Small Open Economy Simulation Methods

This document contains extra information not contained either in the software or in the handout that describes the model solved in the TractableBufferStock problem.

1 Simulation Methods

A difficulty in simulating the small open economy version of the model is that in principle, the economy contains an infinite number of generations of different ages, each of which may have different values of the state variable and a different labor income mass (=population times labor supply times wage). Exact simulation would therefore require an infinite number of data points.

We address this problem by restricting consumers' permissible values of m to a finite number of points; we call these 'classes,' and our simulations approximate the transitions among classes by keeping track of the mass of labor income in each class. Specifically, using a precision parameter Q we map the interval $[1, \infty]$ to a set of points $\vec{\mu} = \{\mu[0], \mu[1], \ldots, \mu[Q]\}$ (including the steady-state $\check{m}$ as one of the $\mu[q]$, and beginning with $\mu[0] = 1$). The mass of labor income for each class is measured by $\mathsf{L}_t[q]$, so that, e.g., the level of aggregate market resources is

$$M_t = \sum_q \mu[q]\mathsf{L}_t[q]. \tag{1}$$

In the 'true' model, a period-t person in class q who survived into period $t+1$ would end up with

$$m_+[q] = (\mu[q] - \mathrm{c}(\mu[q]))\mathcal{R} + 1. \tag{2}$$

However, m_+ is almost certain not to be among the measure-0 set of permissible levels of wealth enumerated in $\vec{\mu}$. We therefore assign a pro-rata proportion of the mass of households who belonged to class q in period t to the nearest-neighbors above and below the 'true' level of m_+ assigned by the model. For example, if m_+ lies 0.75 of the distance between classes q' and $q'+1$, we would assign 75 percent of the mass of surviving households to $\mu[q'+1]$ and 25 percent to $\mu[q']$. Formally, define the function $\underline{\varrho}(m)$ which yields the index for the nearest class below a given m. Defining the 'upper transition weight' $\mathtt{w}_+$ as

$$\mathtt{w}_+[q] = \left(\frac{m_+[q] - \mu[\underline{\varrho}(m_+[q])]}{\mu[\underline{\varrho}(m_+[q]) + 1] - \mu[\underline{\varrho}(m_+[q])]} \right) \tag{3}$$

we allocate proportion $\mathtt{w}_+[q]$ of the mass of $\mathsf{L}_t[q]$ to period-$t+1$ class $\underline{\varrho}(m_+[q])$ and the remainder to class $\underline{\varrho}(m_+[q]) + 1$, so that $(1 - \mathtt{w}_+[q])\mu[\underline{\varrho}] + \mathtt{w}_+[q]\mu[\underline{\varrho} + 1] = m_+$.

The complete set of such weights yields a transition matrix $\mathtt{W}$ that indicates how any given vector of masses in t contributes to the vector of masses in $t+1$.

To complete the specification, we need to describe the effects of population growth and unemployment. Unemployment shrinks the population mass, multiplying it by factor $\cancel{\mho}$. Recall that unemployed consumers move out of the economy and are replaced by newborns who are employed in their first period of life and receive a beginning-of-life transfer (a 'stake'), which may be zero. Normalizing the mass of the newborn generation to 1, we can capture an arbitrary distribution of initial stakes across the classes by the diagonal matrix S the sum of whose diagonals is 1. Thus,

$$\mathsf{L}_{t+1} \;=\; \cancel{\mho}\mathsf{G}\mathtt{W}\mathsf{L}_t + S(\mathsf{G}\Xi)^{t+1} \tag{4}$$

where we assume that initial wages and labor supply in period 0 were $\mathsf{W}_0 = L_0 = 1$.

In this formulation, the size of the population grows in steady state by a factor Ξ per period. It is more useful, however, to work with a normalized version in which the size of each class is normalized by the labor income mass of the newborn generation. Using $\mathcal{L}$ for the normalized (relative) mass, we have

$$\mathcal{L}_{t+1} \;=\; (\cancel{\mho}/\Xi)\mathtt{W}\mathcal{L}_t + S \tag{5}$$