Productivity and the Natural Rate of Unemployment*

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I propose an econometric model that improves upon existing methods of estimating the natural rate of unemployment (NAIRU) by using information contained in the trend of productivity growth. My approach enhances the recently proposed model of Staiger, Stock and Watson (1997) in several respects. Statistically speaking, the method substantially shrinks the width of the 95% confidence interval, performs better in an out-of-sample inflation forecasting exercise, and is more robust to alternative statistical assumptions. In economic terms, the productivity-augmented model generates a more realistic time profile of the NAIRU, and implies estimates of the Phillips curve slope and the sacrifice ratio that are more in line with conventional wisdom. I also test whether the natural rate is correlated with the level or with the change of the productivity growth trend. I find support for the "level" hypothesis in both the US and international data.

Keywords: natural rate of unemployment, productivity, Phillips curve, time-varying parameters, Kalman filter

JEL Classification: C22, E31, E50

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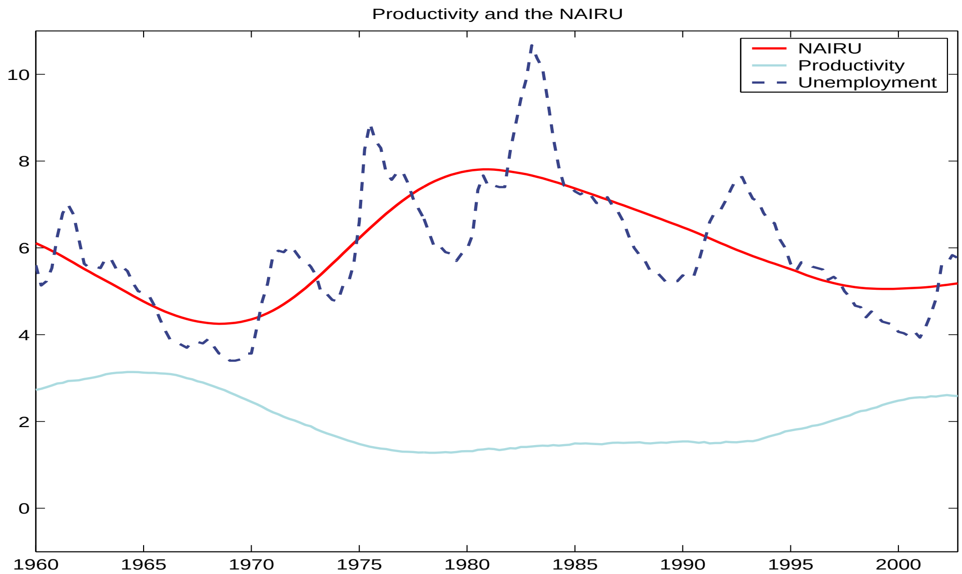
1. INTRODUCTION

Two central features of the natural rate of unemployment (NAIRU, nonaccelerating inflation rate of unemployment) are its substantial time variation and the considerable uncertainty that surrounds it. Furthermore, recent empirical work (Staiger, Stock and Watson, 2001) has found a strong negative correlation between the natural rate and the trend of productivity growth in the United States. This paper proposes an econometric model that improves upon existing method for estimating the NAIRU of Staiger, Stock and Watson (1997) by using information contained in the trend of productivity growth. My method makes it possible to estimate the natural rate more precisely and outperforms current approaches in several other respects.

Many authors (Gordon, 1997, Gordon, 1998, Katz and Krueger, 1998, Staiger *et al.*, 2001, and others) document that the time profile of the natural rate varies substantially over time. For example, Gordon's (1997) preferred estimate of the NAIRU declines from a peak of about 6.5% in 1980 to a low of 5.6% by mid-1996. Besides being of interest for the monetary authority, the estimate of the natural rate is crucial for producing accurate inflation forecasts. The failure to account for the time variation in the natural rate caused the forecasting performance of the standard Phillips curves to break down in the late 1990s (Ball and Moffitt, 2001). Consequently, it is not acceptable to model the natural rate as a constant.

Staiger *et al.* (2001) report that the trends of unemployment and productivity growth co-move strongly. I reproduce their finding in Figure 1. The correlation between the trends in unemployment and productivity growth (as measured by Staiger *et al.*, 2001) over the 1960–2001 period is -0.8.

FIG. 1. Productivity and the Natural Rate of Unemployment (NAIRU) Bandpass



Notes: The trends are estimated using the Baxter and King (1999) bandpass filter with upper cutoff frequency of 60 quarters.

TABLE 1.

Averages for Productivity, Unemployment and Inflation

	1960–1973	**1974–1995**	**1995–2002**
Productivity Nonfarm Business	2.759	2.009	2.286
Unemployment	4.953	5.925	4.869
Inflation	2.818	4.234	2.378
Diff Productivity Nonfarm Business	−0.016	−0.001	0.026

Notes: Quarterly Data. The productivity means are calculated from the productivity trend generated by the Baxter and King (1999) bandpass filter with upper cutoff frequency of 60 quarters.

The descriptive statistics for productivity and unemployment displayed in Table 1 also illustrate this inverse relationship. Productivity growth was rapid before 1973, slowed down after 1973 for more than twenty years and then resumed vigorously after 1995. The average unemployment rate, on the other hand, was more than 1 percentage point higher between 1973 and 1995 than before and after that period. The co-movement of unemployment and productivity growth trends is an impressive result since no unemployment data are used to construct productivity data (and vice versa).

This paper extends the random walk framework (Staiger et al., 1997) by using information contained in the trend of productivity growth. The original formulation assumes that the natural rate is completely driven by an unobserved white noise variable. The productivity growth trend explains a large fraction of the variation in the NAIRU and including it significantly shrinks the unexplained part. This finding is intuitive since including a relevant variable in the regression usually improves its explanatory power.

My approach improves upon the random walk method in several respects. Statistically speaking, the method substantially shrinks the width of the 95% confidence interval, performs better in an out-of-sample inflation forecasting exercise, and is more robust to alternative statistical assumptions. In economic terms, the productivity-augmented model generates a more realistic time profile of the NAIRU, and implies estimates of the Phillips curve slope and the sacrifice ratio that are in line with conventional wisdom.

I also test whether the natural rate is correlated with the level or with the change of the productivity growth trend. I find support for the "level" hypothesis in both the US and international data. This is surprising because many models proposed recently to explain the relationship between the natural rate and productivity growth (Meyer, 2001, Ball and Moffitt, 2001, Mankiw and Reis, 2003) are consistent with the "change" hypothesis. A crucial assumption of this recent research is that workers' estimates of productivity growth adjust slowly to true productivity growth. As a result, these models explain the negative correlation between the natural rate and the change in productivity growth, rather than between the natural rate and the level of productivity growth. However, there is some theoretical work in the job search and matching literature that might be able to explain a negative correlation of the NAIRU and the level of productivity growth.

The paper is organized as follows. Section 2 reviews the theoretical literature on the relationship between the natural rate and productivity. Section 3 proposes the econometric model and discusses econometric issues. Section 4 reports the empirical findings of the baseline model for the US. Section 5 summarizes the robustness of the results and tests the “level vs. change” hypothesis. Section 6 focuses on the international evidence on the relationship between productivity growth and the natural rate. Section 7 concludes.

2. PRODUCTIVITY AND THE NAIRU: THEORY REVIEW

There are two lines of research that attempt to explain the inverse relationship between productivity growth and the natural rate. Some economists argue that the link is caused by a mismatch between the perceptions of productivity growth by workers and firms. Other explanations, based on search and matching models, propose that productivity growth translates into structural change that also raises unemployment.

In the first line of research, firms are typically assumed to observe the productivity growth trend. Workers, in contrast, have to infer the productivity trend based on limited information. For instance, Braun (1984) and Meyer (2001) assume that workers base their wage claims on a *real-time* estimate of the productivity trend. Ball and Moffitt (2001) suggest that workers' real wage targets depend on *aspirations*, a weighted average of past real wages. Mankiw and Reis (2003) propose a *sticky-information* model in which a randomly chosen fraction of workers updates information on productivity each period. While these models start from different premises, they have similar implications. They all predict that an increase in productivity growth *temporarily* lowers inflation and the natural rate. Strictly speaking, these models do not explain the correlation between the levels of the NAIRU and of productivity growth. However, if the workers' estimates of the productivity growth trend adjusts slowly to the true value, the implications of these models will be hard to distinguish from the level hypothesis.

There is a modest amount of work on the effect of productivity growth on unemployment in the theoretical job search literature (Aghion and Howitt, 1994, Mortensen and Pissarides, 1998). Productivity growth has two competing effects. First, higher labor productivity growth increases the value of a worker to the firm, and stimulates the creation of job vacancies. This, in turn, causes unemployment to decline (the capitalization effect). Second, higher productivity growth is often accompanied by structural change. Old jobs are destroyed and replaced by new ones (the creative destruction effect). As a result, productivity acceleration shortens employment duration and raises the natural rate. Consequently, the correlation between productivity growth and the natural rate depends on the relative size of these two effects.

Empirically, the negative correlation between the productivity growth trend and the natural rate in Figure 1 along with the results below suggest that the capitalization effect is stronger.

3. ECONOMETRIC MODEL

In the empirical literature, the NAIRU is typically estimated in the Phillips curve framework as the rate of unemployment that is consistent with stable inflation expectations. This section first reviews existing methods of modelling the natural rate both as a constant and in the time-varying parameter framework. I then propose the productivity-augmented model and discuss some econometric issues.

Assume for the moment that the natural rate $\bar{u}$ is constant. To estimate the NAIRU, start with the expectations-augmented Phillips curve,

$$\Delta\pi_t = \gamma(L)(u_{t-1} - \bar{u}) + \delta(L)\Delta\pi_{t-1} + \alpha(L)X_t + \varepsilon_t, \tag{1}$$

where $\gamma(L), \delta(L)$ and $\alpha(L)$ are lag polynomials and X_t includes supply shocks. The Phillips curve (1) follows much of the empirical literature in assuming that inflation expectations follow a random walk, $\pi_t^e = \pi_{t-1}$. The natural rate can be estimated by ordinary least squares (OLS) as the horizontal intercept. Specifically, after running the regression

$$\Delta\pi_t = \gamma_0 + \gamma(L)u_{t-1} + \delta(L)\Delta\pi_{t-1} + \alpha(L)X_t + \varepsilon_t$$

the estimate of the NAIRU is $\bar{u} = -\gamma_0/\gamma(1)$, where $\gamma(1)$ is the sum of unemployment coefficients.

The constancy of the natural rate is a very restrictive assumption. As Gordon (1997, p. 12) puts it, “the NAIRU is not carved in stone.” Friedman (1968) defines the natural rate as the "level which would be ground out by the Walrasian system of general equilibrium equations, provided there is imbedded in them the actual structural characteristics of the labor and commodity markets." To capture the effects of changes in these characteristics on the NAIRU, Staiger, Stock and Watson (1997) propose the time-varying parameter model (Kalman filter),

$$\begin{aligned}\Delta\pi_t &= \gamma(L)(u_{t-1} - \bar{u}_{t-1}) + \delta(L)\Delta\pi_{t-1} + \alpha(L)X_t + \varepsilon_t,\\ \bar{u}_t &= \bar{u}_{t-1} + \eta_t, \qquad \operatorname{var}(\eta_t) = \lambda \operatorname{var}(\varepsilon_t).\end{aligned} \tag{2}$$

The natural rate $\bar{u}_t$ is now assumed to follow a random walk. The variation in $\bar{u}_t$ is governed by the signal-to-noise parameter $\lambda \equiv \operatorname{var}(\eta_t)/\operatorname{var}(\varepsilon_t)$. If $\lambda = 0$, the NAIRU is constant and (2) reduces to (1).

The random walk model is a flexible device that captures the unobserved time-variation in the natural rate. However, when there are variables that are informative about the NAIRU, it is more efficient to include them in the model. This decreases the unexplained variation, $\operatorname{var}(\eta_t)$ and increases .

The Kalman filter framework can be generalized by including exogenous variables Z_t in the second equation of (2),

$$\begin{aligned}\Delta\pi_t &= \gamma(L)(u_{t-1} - \bar{u}_{t-1}) + \delta(L)\Delta\pi_{t-1} + \alpha(L)X_t + \varepsilon_t,\\ \bar{u}_t &= \bar{u}_{t-1} + \beta^\top \Delta Z_t + \eta_t, \qquad \operatorname{var}(\eta_t) = \lambda \operatorname{var}(\varepsilon_t).\end{aligned} \tag{3}$$

In model (3) a fraction of the variation in the state variable $\bar{u}_t$ is explained by exogenous variables in Z_t. Consequently, the variance of the error term η_t in the random walk model (2) is greater than the variance of the error in (3) and as a result model (3) explains $\bar{u}_t$ better.

The natural rate in (3) is modelled as a random walk driven by the exogenous variables Z_t and the error term η_t. This specification is chosen, as is usual in the literature to allow for persistent deviations of $\bar{u}_t$ from βZ_t. It is important to note that the specification (3) implies that differences in Z_t affect differences in the natural rate $\bar{u}_t$ or, equivalently, that levels of Z_t affect levels of $\bar{u}_t$.

In the baseline specification the exogenous variables Z_t consist of the productivity trend θ_t^*, obtained by the Kalman filter as explained below. Specification (3) assumes that Z_t only influences $\bar{u}_t$. Therefore, there is no direct effect of Z_t on $\Delta\pi_t$. Supply shocks X_t are, in contrast, very volatile and I therefore follow the existing literature in assuming that they do not affect the natural rate $\bar{u}_t$.

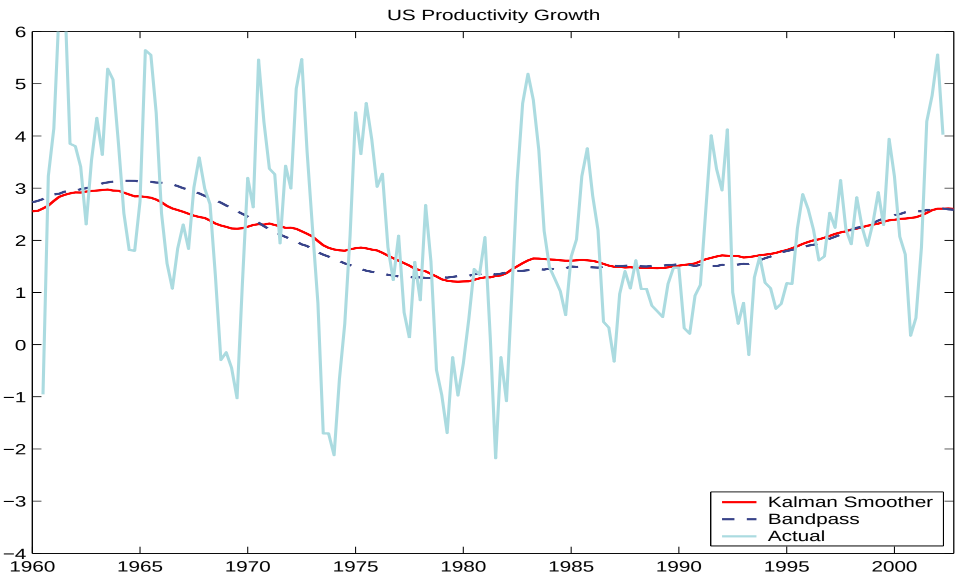
Econometric Issues

The productivity trend θ_t^* is estimated by the random walk plus noise (or local level) model,

$$\theta_t = \theta_t^* + z_{Tt}, \qquad \theta_t^* = \theta_{t-1}^* + z_{Pt}, \qquad \mathrm{var}(z_{Tt}) = \lambda_\theta \mathrm{var}(z_{Pt}) \tag{4}$$

where θ_t is the observed, measured productivity growth rate, θ_t^* is the unobserved trend to be estimated and z_{Tt} and z_{Pt} are the temporary and permanent shocks to productivity, respectively. This specification is a flexible device that makes it possible to extract the long-run trend from the time series using the Kalman filter algorithm. The Kalman filter is an alternative to the more common filters, such as the Hodrick–Prescott filter. The advantage of the Kalman filter model (4) is that the algorithm produces

FIG. 2. Productivity Growth and Trend



Notes: The trends are estimated using the Baxter and King (1999) bandpass filter with upper cutoff frequency of 60 quarters and Kalman smoother with the signal-to-noise ratio $\lambda_\theta = 0.005$. The actual productivity growth is year-on-year quarterly growth.

an optimal estimator of the trend (the minimum mean squared error linear estimator), see for example Harvey (1989).[1]

I assume that the disturbances ε_t and η_t in (2) and (3) are i.i.d. normal $\mathcal{N}(0, \text{var}(\varepsilon_t))$ and $\mathcal{N}(0, \text{var}(\eta_t))$, respectively. Furthermore, the disturbances ε_t and η_t are also assumed to be uncorrelated. I estimate the parameters $\{\gamma(L), \delta(L), \alpha(L), \beta, \text{var}(\varepsilon_t)\}$ by maximum likelihood (ML), as described in Harvey (1989).

The amount of time variation in $\bar{u}_t$ is governed by the signal-to-noise parameter λ. Since the NAIRU varies slowly over time, the variance of η_t is usually very small. Consequently, the estimate of $\text{var}(\eta_t)$ has bad small-

[1]I consider the productivity trend θ_t^* obtained by the bandpass filter in section 5 below. Figure 2 compares the productivity trends measured by the Kalman and bandpass filters.

sample properties—it is estimated very imprecisely, with a downward bias. Besides, in small samples the distribution of the signal-to-noise ratio λ has a non-zero probability mass at zero, a so-called pile-up problem. This results in the implied natural rate of unemployment being too smooth, often almost constant. Consequently, I follow existing literature (Staiger *et al.*, 2001, King, Stock and Watson, 1995, and others) in imposing a reasonable value for λ instead and estimating the remaining parameters by ML. Interestingly, the estimate of the natural rate in the productivity model (3) is considerably more robust to the choice of λ than in the random walk model, as documented in section 5.

Stock and Watson (1998) propose an alternative to imposing λ. The method consists of conducting the sup-Wald structural break test for a break in the constant in the Phillips curve. One then compares the test statistic to the table of Stock and Watson (1998) critical values and retrieves the implied median-unbiased estimate of λ together with its confidence intervals. I estimate the signal-to-noise ratios λ using this method and report the median-unbiased estimates of $\mathrm{var}(\eta_t)$ in the last line of Table 2 below. However, I do not use the method in the calculations because the confidence intervals for λ tend to be very wide and the estimated signal-to-noise ratios are less satisfactory in some cases than the imposed ones.

4. EMPIRICAL RESULTS

In this section I compare three benchmark models estimating the natural rate: the constant NAIRU model (1), the random walk model (2) of Staiger *et al.* (1997), and the productivity-augmented model (3). The major flaw

TABLE 2.

Estimation Results, Baseline Models

	OLS	**Random Walk**	**Productivity**
Sum of Coeffs on Unemployment	−0.199	−0.147	−0.224
Std Error on Sum of Unemployment	0.076	0.111	0.121
P value on Lags of Unemployment	0.009	0.000	0.000
P value on Lags of Inflation	0.000	0.000	0.000
P value on Supply Shocks	0.004	0.285	0.024
P value on Productivity	–	–	0.062
Coefficient on Productivity	–	–	−2.251
Mean Width of Confidence Intervals	3.078	4.114	2.985
Sacrifice Ratio	2.297	2.979	2.101
Estimate of the Signal-to-Noise Ratio	–	0.011	0.022
Log-likelihood	–	−127.880	−129.380

Notes: The confidence intervals for the OLS model are calculated following Staiger *et al.* (1997)using the Anderson–Rubin exact method based on inverting the F statistic of H_0: $\bar{u} = u_0$ for various values of u_0. All p values are based on the White heteroscedasticity-robust standard errors. P value of 0 means less than 5×10^{-4}.

of the first model is the assumption of the constant NAIRU. The random walk model performs poorly in several respects. It produces wide confidence intervals and an unrealistic time profile of the natural rate. Moreover, the slope of the Phillips curve and the implied sacrifice ratio are not in line with the conventional wisdom. The productivity model alleviates these shortcomings.

Table 2 reports the findings. Column one summarizes the traditional backward-looking Phillips curve with the constant NAIRU. Its principal strength is that the statistics are in line with conventional wisdom. The lags of inflation, unemployment, and supply shocks are significant. The value of the slope, $\gamma(1)$, is comparable to the findings of other authors. Finally, the implied sacrifice ratio, the unemployment cost of reducing inflation, is in the upper range of estimates obtained by Ball (1994) and others. In light of the recent decline of the natural rate, its assumed constancy is a crucial shortcoming. The reported estimate of the natural rate of about 6% can be in principle interpreted as the average value of the true time-varying NAIRU (TV-NAIRU). However, it is questionable how useful it is for the monetary authority to know the average natural rate when the NAIRU varies substantially.

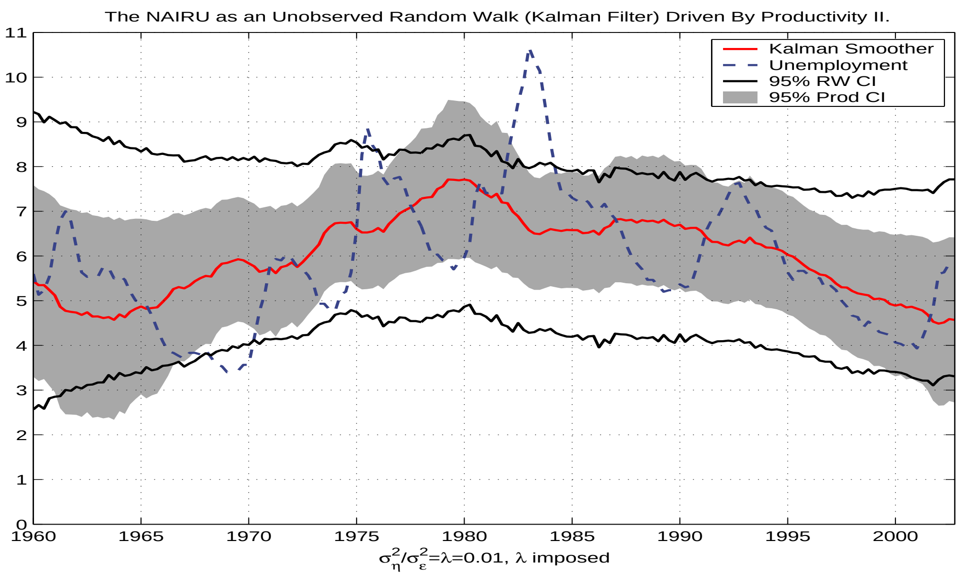
4.1. Confidence Intervals

The random walk model (2) was proposed to account for the time-variation in the natural rate. However, it is problematic in other respects. First, the slope of the random walk Phillips curve in column 2 of Table 2 is considerably smaller in magnitude than the slope of the OLS Phillips curve and statistically insignificant.[2] This has a crucial implication for the estimate of the natural rate. The slope $\gamma(1)$ enters the denominator of the estimate of the NAIRU, which causes the natural rate to be unidentified when the slope is zero. Similarly, when $\gamma(1)$ is small the confidence intervals for the natural rate tend to be extremely wide (see Staiger *et al.*, 2001). The productivity model, in contrast, implies a greater Phillips curve slope, which narrows the NAIRU confidence intervals. This subsection compares the confidence intervals implied by the various models.

Figure 3 depicts the NAIRU confidence intervals implied by the random walk and the productivity models. The confidence intervals are calculated from the variance of the Kalman smoother estimate of $\bar{u}_t$ with a delta method

[2]The estimates of slopes of the OLS and random walk Phillips curves are consistent with other specifications in the literature, e.g. Staiger *et al.* (1997) and Staiger *et al.* (2001), respectively.

FIG. 3. Comparison of Productivity-driven and Random Walk Confidence Intervals for the NAIRU

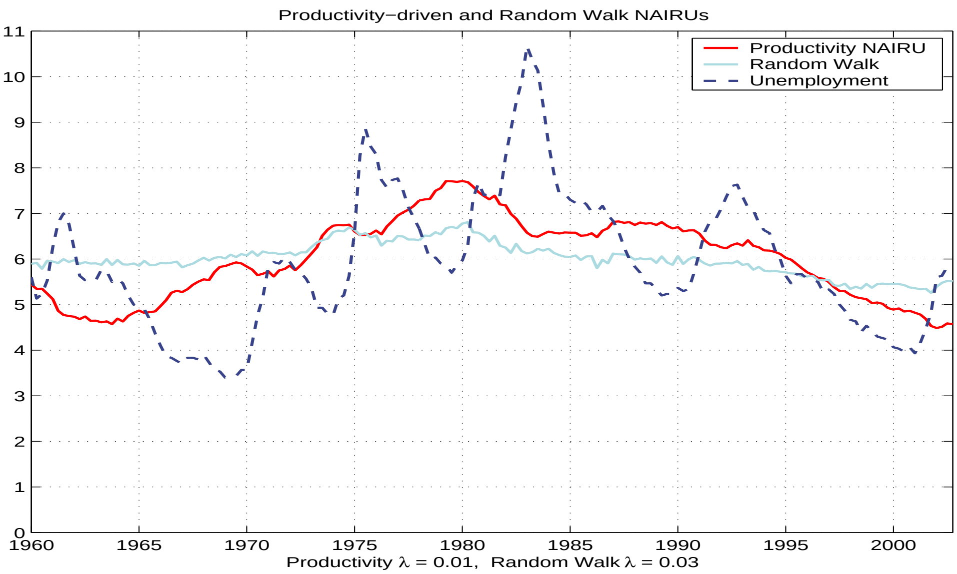


Notes: The natural rate of unemployment is estimated by the Kalman filter model (3) with the signal-to-noise ratio $\lambda = 0.01$. The confidence intervals have 95% size and are obtained from the estimate of the variance of the Kalman smoother and corrected for parameter uncertainty following Ansley and Kohn (1986).

correction for parameter uncertainty due to Ansley and Kohn (1986). The method is consistent with Staiger *et al.* (1997).

The average width of confidence intervals shrinks from 4.1 percentage points for the random walk model to 3.1 percentage points with the productivity model. The black solid line in Figure 3 depicts the replication with quarterly data, 1960–2002 of the 95% confidence intervals of Staiger *et al.* (1997). In fact, even though the point estimates of the natural rates in Figure 4 differ by up to 1%, the shaded confidence band for the productivity model is for most periods within the confidence band of the random walk model.

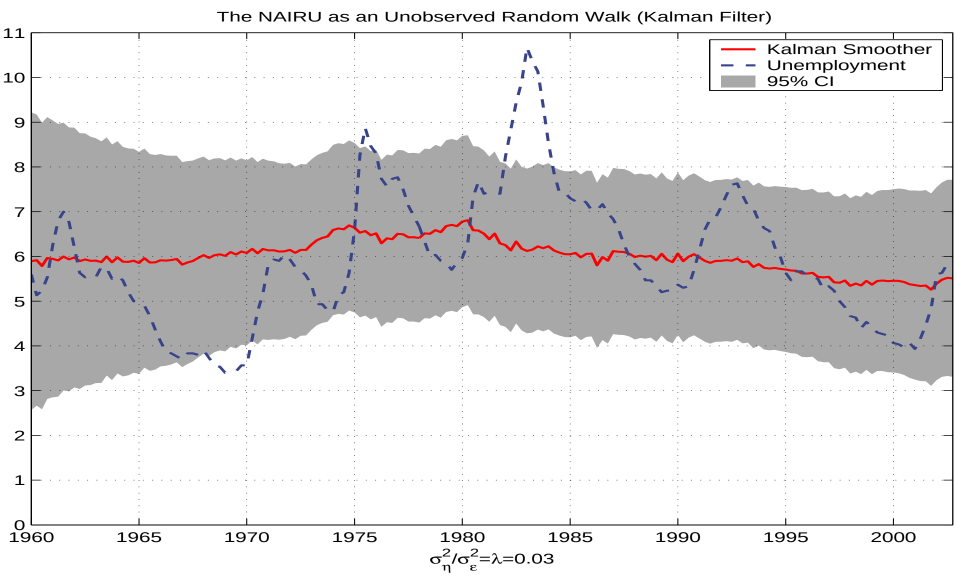
FIG. 4. Comparison of Productivity-driven and Random Walk Natural Rates of Unemployment



Notes: The random walk natural rate of unemployment is estimated by the Kalman filter model (2) with the signal-to-noise ratio $\lambda = 0.01$. The productivity natural rate of unemployment is estimated by the Kalman filter model (3) with the signal-to-noise ratio $\lambda = 0.01$.

A major problem of model (2) is that the time variation natural rate $\bar{u}_t$ is driven exclusively by the white noise η_t. This is a reasonable solution when one is agnostic about the possible causes for the movements of the NAIRU. However, when we have candidates that are plausibly correlated with the NAIRU, it is beneficial to use the additional information. If the correlation between the natural rate and these variables Z_t is strong enough, adding them to the econometric model increases the quality of the estimated natural rate and the parameters. Intuitively, including a relevant explanatory variable in the regression improves the precision of the estimates.

FIG. 5. Random Walk Natural Rate of Unemployment



Notes: The natural rate of unemployment is estimated by the Kalman filter model (2) and assumed to follow unobserved random walk model with the signal-to-noise ratio $\lambda = 0.03$. The parameter λ is chosen to mimic the estimates of Staiger *et al.* (1997). The confidence intervals have 95% size and are obtained from the estimate of the variance of the Kalman smoother and corrected for parameter uncertainty following Ansley and Kohn (1986).

4.2. Time Profile of the Estimates of the Natural Rate

One important shortcoming of the random walk model is that it implies an unrealistic estimate of the time profile of the natural rate. There is not only evidence that the NAIRU is not constant, we actually have a prior on how it varies. We typically think of it as a slowly varying, smooth function of time. Large abrupt changes in the natural rate are very unlikely.

The NAIRU time profile of the random walk model is displayed in Figure 5, a replication of Staiger's *et al.* (1997) Figure 6. There are at least two problems with the NAIRU profile: it is both excessively sensitive and excessively smooth. More precisely, there is too much high-frequency variation and not enough low-frequency variation in the estimate of the natural rate. The natural rate of Figure 5 is not very smooth, at the same time its constancy cannot be rejected. Unfortunately, increasing the λ parameter affects the high-frequency variation in the natural rate and does not improve the results much.[3] The random walk model substitutes the lack of low-frequency variation in the natural rate with the high-frequency variation. Figure 4 documents that this does not work satisfactorily. Both the rise in the NAIRU in the late 1970s and its fall in the late 1990s are much less pronounced for the random walk model than for the productivity model.

Interestingly, the shape of the time-varying NAIRU implied by the productivity model is much closer to the conventional wisdom. This is because the productivity growth adds more low-frequency variation and at the same time decreasing λ makes it possible to lower the high-frequency variation in the NAIRU. The productivity growth is borderline significant with a p value of 0.048. The sensitivity of the natural rate with respect to the productivity growth, β, is about -2, which means that if the level of productivity growth increases by 1%, the natural rate declines by 2 percentage points. Assuming productivity growth went up by 0.6 percentage points in the late 1990s, this translates into a 1.2 percentage points fall in the NAIRU, as is also documented in Figure 4.

4.3. Slope of the Phillips Curve and the Sacrifice Ratio

I note above that using the information from the productivity growth trend increases the magnitude of the Phillips curve coefficient and its signif-

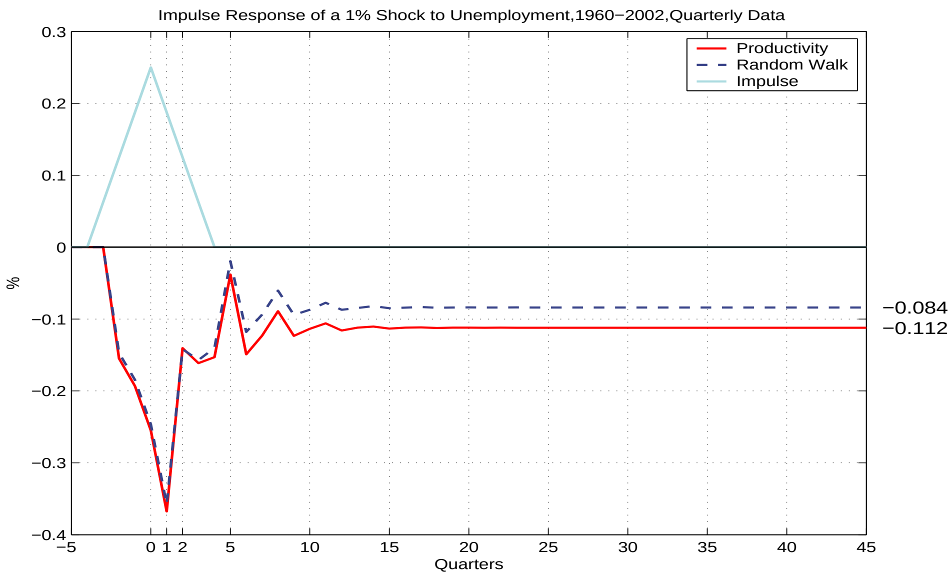
[3] I explore the effects on the estimates of the natural rate of imposing other values of λ in subsection 5.2 below.

icance. The first row of Table 2 documents this finding. The slope implied by the random walk model is substantially smaller and considerably less significant than the slopes of the OLS and productivity models.

The magnitude of the slope of the Phillips curve determines the sacrifice ratio, the unemployment cost of decreasing inflation by one percentage point. The sacrifice ratio is estimated from the Phillips curve as the long-run response of inflation π_t to a one percentage point increase in the unemployment rate over one year. To get the intuition, suppose one has the Phillips curve with no inflation lags on the right-hand side. The long-run response of inflation to a one percentage point increase in unemployment over a one year period is the sum of the unemployment coefficients $\gamma(1)$. Equivalently, an increase in unemployment by $|1/\gamma(1)|$ percentage points results in a 1 percentage point decline in inflation rate.

Figure 6 compares the long-run inflation responses to a 1% unemployment shock for the productivity and random walk models. As already suggested by the slopes of the Phillips curves, the long-run response of the productivity model is about 30% bigger than that of the random walk model, -0.11 vs. -0.08. This translates to different sacrifice ratios, as documented by second last line of Table 2. The estimate of the sacrifice ratio implied by the random walk model is substantially higher than the estimates from the OLS and productivity models. Assuming a coefficient of 2 in Okun's law, the output cost of disinflation is about 6 for the random walk model and about 4.5 for the productivity model. Ball's (1994) estimates of sacrifice ratios for the disinflation episodes in the OECD countries generally range between 0 and 4. Consequently, the sacrifice ratio implied by the random walk model

FIG. 6. Comparison of the Implied Inflation Responses to a 1% Shock to Unemployment



seems too high. In contrast, the sacrifice ratio implied by the productivity model is more in line with the conventional wisdom.

4.4. Forecasting

It is standard to use the Phillips curve as an inflation forecasting tool. To produce h-period ahead inflation forecasts the following modification of the Phillips curve (1) is often used,

$$\Delta_h \pi_t = \gamma(L)(u_{t-1} - \bar{u}_{t-1}) + \delta(L)\Delta\pi_{t-1} + \varepsilon_t, \tag{5}$$

where $\Delta_h \pi_t = \pi_{t+h} - \pi_t$ is the h-period change in inflation. Stock and Watson (1999) argue that the Phillips curve (5) generates more accurate one-year-ahead inflation forecasts than the majority of other relationships.

TABLE 3.

Out-of-Sample and In-Sam-le Forecasts, MSEs Relative to the Constant NAIRU MSE

	Out-of-Sample		**In Sample**	
Horizon h (quarters)	**Prod**	**RW**	**Prod**	**RW**
1	0.991	1.101	0.975	0.926
2	0.915	0.928	1.043	1.098
3	0.918	0.948	0.978	0.994
4	0.876	0.921	0.958	0.996
8	0.857	0.942	0.894	0.951
12	0.876	0.934	0.924	0.955
Mean	0.906	0.962	0.962	0.986

Notes: The out-of-sample results are based on the rolling regressions with increasing window and fixed initial date, 1960–2002.

To evaluate the quality of the two alternative estimates of the natural rate, $\bar{u}_{t,1}$ and $\bar{u}_{t,2}$, I employ the following procedure. Given $\bar{u}_{t,i}$ and inflation and unemployment data I estimate the regression (5) and produce both out-of-sample and in-sample inflation forecasts. The out-of-sample forecasts are generated by rolling regressions that are recursively estimated based on variables dated time $1, \ldots, t$. Because it is first necessary to use the information in the whole sample $1, \ldots, T$ to estimate the NAIRU, $\bar{u}_t$, these regressions should not be interpreted as a real-time procedure. However, the method is still valid for evaluation of the quality of alternative NAIRU estimates.[4] As an alternative to the out-of-sample procedure one can produce the forecasts in an in-sample framework as fitted values from regression (5) based on the information $1, \ldots, T$.

[4]One can in principle imagine implementing this procedure in a real-time-like framework and estimating the models (2) or (3) at each time period t. However, because there is much of uncertainty about the natural rate at the end of the sample, this would probably produce extremely noisy inflation forecasts and is not pursued here.

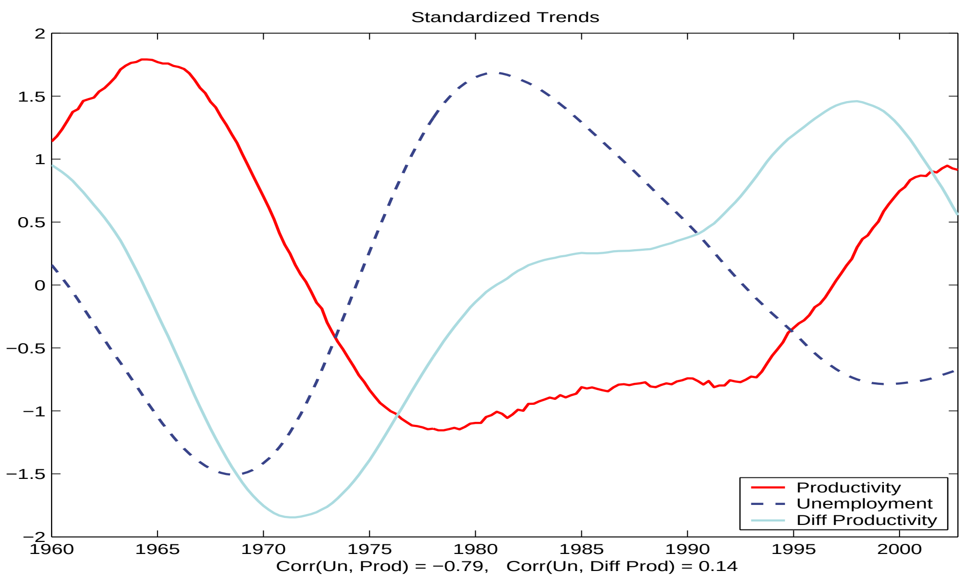
Table 3 displays the mean squared errors (MSE) of the forecasts of the productivity and random walk models relative to the MSE of the constant NAIRU for various forecasting horizons h. The out-of-sample forecasts of the productivity model are on average 9% better than the constant NAIRU forecasts and 5% more precise than the random walk forecasts. The differences are more pronounced at longer forecasting horizons. This is because the slopes of the Phillips curve are greater for longer horizons. This makes sense since when unemployment is above the NAIRU one would expect inflation to steadily increase. As a result, $\Delta_h \pi_t \approx h \times \Delta_1 \pi_t$. The right panel of Table 3 displays the in-sample results. The differences in quality of the various models are not as significant as in the out-of-sample case. However, the productivity model still performs best and the constant model does relatively poorly.

Overall, accounting for the time-variation in the natural rate results in more precise inflation forecasts. These forecasts are further improved by using the information about productivity growth.

5. SPECIFICATION TESTING AND ROBUSTNESS

This section considers various issues in specification testing. I first test whether the natural rate is correlated with *the level or with the change in productivity growth.* I then focus on the choice of the signal-to-noise ratio λ. Finally, I investigate whether my findings from the previous sections hold for alternative productivity, unemployment, inflation, inflation expectations series.

FIG. 7. Standardized Trends in Unemployment, Productivity and Change in Productivity



5.1. Changes or Levels?

The previous sections investigate the relationship between the *level* of the NAIRU and the *level* of productivity growth. However, most theoretical models that address the issue imply, a correlation between the *level* of the natural rate and the *change* in productivity growth. I now focus on this relationship.

Informally, the last row of Table 1 suggests that the relationship between the change in productivity growth and the NAIRU is empirically not as strong as between the level of productivity growth and the NAIRU. The average change in productivity growth was small during 1960–1973, larger in 1974–1995, and still larger after 1995. Unemployment on the other hand was low before 1973 and after 1995 and high between 1974 and 1995. The

TABLE 4.

Estimation Results, Difference vs. Level of Productivity

	Diff Model	**Level and Diff Model**	
Sum of Coeffs on Unemployment	-0.169	-0.202	
Std Error on Sum of Unemployment	0.101	0.115	
P value on Lags of Unemployment	0.000	0.000	
P value on Lags of Inflation	0.000	0.000	
P value on Supply Shocks	0.021	0.030	
Coefficients		Level	Diff
P value on Productivity	0.441	0.064	0.6075
Coefficient on Productivity	-31.529	-1.876	-18.754
Mean Width of Confidence Intervals	3.866	–	
Sacrifice Ratio	2.686	2.349	
Estimate of the Signal-to-Noise Ratio	0.030	0.000	

Notes: All p values are based on the White heteroscedasticity-robust standard errors. P value of 0 means less than 5×10^{-4}.

first column in Table 6 below displays the correlations between productivity and unemployment trends in the US. The correlations between the changes in productivity growth $\theta^*_{t+h} - \theta^*_t$ and unemployment trend are often positive and tend to be negative only for very long horizons ($h = 7$ years and longer). The correlation between the levels of productivity trend and the NAIRU, in contrast, is high and negative, -0.81. Finally, Figure 7 displays the trends in unemployment, productivity, and productivity growth standardized to have a zero mean and unit variance. The figure confirms that the correlation between the change in productivity growth and the natural rate has the wrong sign. In particular, in the 1970s unemployment was rising, productivity growth was falling and yet the change in productivity growth was increasing.

To obtain more rigorous evidence I estimate model (3) with the change in the productivity trend as an exogenous variable, $Z_t = \Delta\theta^*_t$. The first column of Table 4 summarizes this case. This model does not improve on the random walk model. While the coefficient on the productivity variable $\Delta\theta_t^*$ is quite high (because the difference varies less than the level), it is insignificant. The confidence intervals for the natural rate are almost as wide as with the random walk model and the sacrifice ratio is very high.

The second column of Table 4 shows the findings for the model with the exogenous variable consisting of both the productivity level and change, $Z_t = (\theta_t^*, \Delta\theta_t^*)^\top$. The change in productivity growth is again insignificant. Other than that the implications of this model are similar to those of the baseline productivity model in Table 2. The value of the coefficient on the productivity level, θ_t^*, is -1.9, the slope of the Phillips curve is greater than in the first column, and the sacrifice ratio smaller.[5]

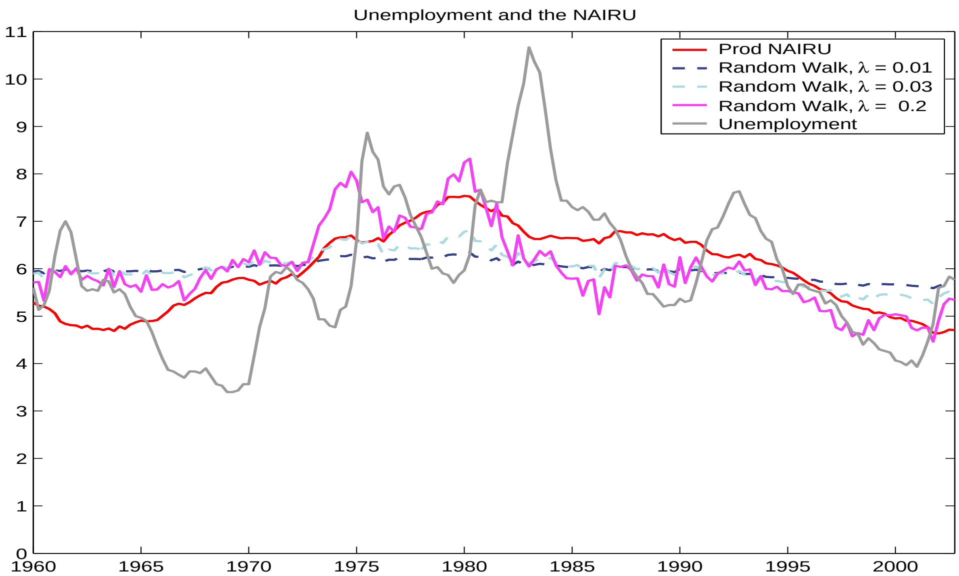
On the whole, both simple correlations and the more rigorous Kalman filter model (3) support the "level" rather than "change" hypothesis. One interpretation is that this finding contradicts the implications of recent models proposed to explain the relationship between productivity and the natural rate (Ball and Moffitt, 2001, Mankiw and Reis, 2003). However, the evidence for the "level" hypothesis may instead suggest that workers update their estimate of the productivity trend very slowly.

5.2. Signal-to-Noise Ratios

In the previous computations I follow much of the literature in imposing the signal-to-noise ratio λ, as opposed to estimating it. The size of λ determines the high-frequency variation in the natural rate. The ideal

[5]One reason why the change in productivity growth $\Delta\theta_t^*$ does not perform well is that it is relatively volatile. However, the results still hold even after filtering $\Delta\theta_t^*$.

FIG. 8a. Comparison of Various Signal-to-Noise Ratios, Random Walk Model



signal-to-noise ratio is big enough for the implied natural rate to capture the time variation and at the same time small enough for the NAIRU to be smooth. I now investigate the sensitivity of the NAIRU time profiles to the choice of the signal-to-noise ratio.

Figure 8a compares the estimates of the natural rates for various λs in the random walk model. This model is sensitive to the choice of λ. Unfortunately, none of the λs delivers the shape generated by the productivity model. The problem is that the choice of λ affects the high-frequency variation rather than the low-frequency variation in $\bar{u}_t$. Consequently, small values of the signal-to-noise ratio imply a smooth but almost constant estimate of the NAIRU. In contrast, a large λ generates a volatile natural rate which fails to capture the smoothness.

FIG. 8b. Comparison of Various Signal-to-Noise Ratios, Productivity Model

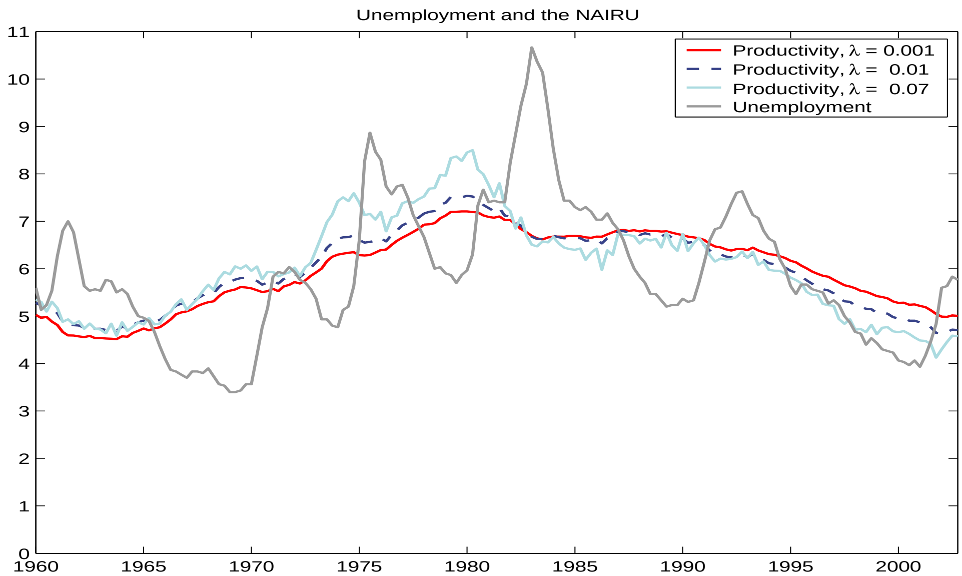


Figure 8b displays the effect of changing λ for the productivity model. Because the productivity variable soaks up much of the time-variation in the NAIRU, the results are robust to the choice of λ. The estimates of the natural rate look very similar for different values of λ. This is a reassuring finding for the productivity model.

5.3. Alternative Time Series

This subsection discusses the implications of the productivity model with alternative productivity, unemployment, inflation, and inflation expectations series. A broad conclusion is that the results reported in section 4 continue to hold.

The first column of Table 5 and and the first panel of Figure 9 summarize the findings for an alternative inflation expectations series. The inflation

FIG. 9. Alternative Time Series

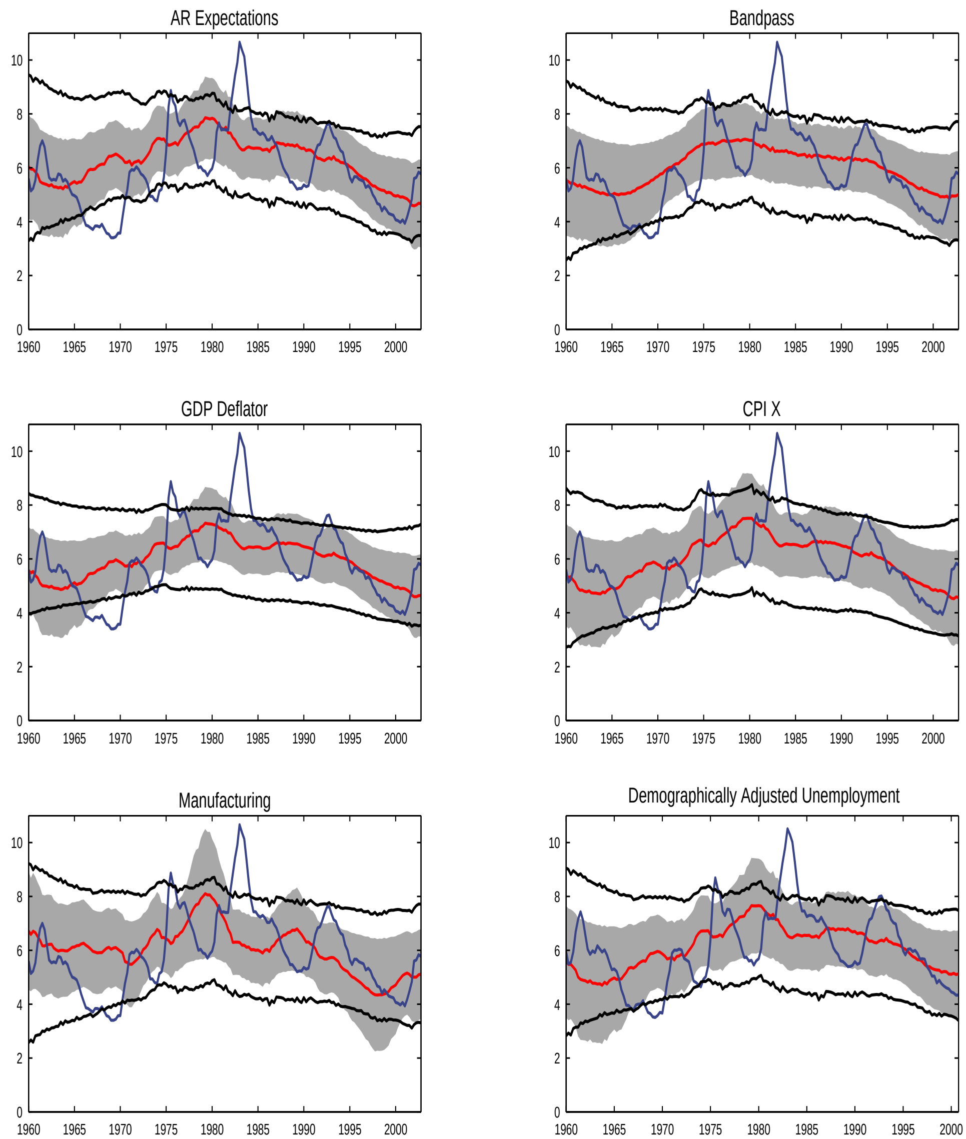


TABLE 5.

MLE Estimation Results, Alternative Time Series

	Base	**ARExp**	**Bpass**	**Mnfctrg**	**GDPD**	**CPI X**	**DemAdjUn**
Sum of Coeffs on Unemployment	-0.213	-0.277	-0.250	-0.212	-0.201	-0.227	-0.218
Std Error on Sum of Unemployment	0.116	0.118	0.131	0.097	0.087	0.133	0.116
P value on Lags of Unemployment	0.000	0.000	0.000	0.000	0.000	0.000	0.000
P value on Lags of Inflation	0.000	0.000	0.000	0.000	0.000	0.000	0.000
P value on Supply Shocks	0.026	0.018	0.027	0.021	0.017	0.230	0.031
P value on Productivity	0.049	0.033	0.133	0.114	0.039	0.074	0.056
Coefficient on Productivity	-1.944	-1.821	-1.159	-2.384	-1.582	-1.688	-1.944
Mean Width of Confidence Intervals	3.083	2.699	2.808	3.155	2.423	2.846	3.012
Sacrifice Ratio	2.224	1.325	1.882	2.341	2.437	2.007	2.206
Estimate of the Signal-to-Noise Ratio	0.006	0.000	0.004	0.004	0.000	0.004	0.004
Log-likelihood	-128.900	-138.330	-130.170	-129.280	-78.157	-121.820	-123.230

Notes: All p values are based on the White heteroscedasticity-robust standard errors. P value of 0 means less than 5×10^{-4}.

expectations were generated as inflation forecasts from an AR(4) process in $\Delta\pi_t$. Interestingly, this model performs even better than the baseline model. Both the Phillips curve slope and the productivity variable are significant. The mean width of confidence intervals for the natural rate shrinks to 2.7%, and the implied sacrifice ratio in terms of GDP is $2 \times 1.3 = 2.6$.

The second column reports the results for an alternative measure of productivity trend, the bandpass filter (see also Figure 2). The confidence intervals shrink considerably again, to 2.8% on average. The sacrifice ratio, $2 \times 1.9 = 3.8$, is in the reasonable range.

The third column describes the implications of model (3) with productivity measured as productivity in manufacturing sector, instead of in the non-farm business sector. The model reduces the sacrifice ratio and increases the magnitude of the Phillips curve slope. Productivity in manufacturing is not a preferred measure of productivity because manufacturing is a relatively small fraction of the economy. Not surprisingly, it turns out that the correlation between this productivity measure and the NAIRU is not as high as in the case of non-farm business sector productivity. Consequently, the productivity variable is not significant. However, the model does a good job at reducing the confidence intervals and obtaining the intuitive time profile of the NAIRU.

The fourth column collects the findings for the GDP deflator as a measure of inflation. These results mimic the implications of the baseline model. The slope of the Phillips curve is significant and the NAIRU confidence intervals are narrow. The sacrifice ratio of $2 \times 2.5 = 5$ is still considerably lower than the random walk sacrifice ratio.

Inflation in the next column is measured by the CPI excluding food and energy. The findings are again similar to the baseline model. The natural rate confidence intervals are narrow, 2.8%. The sacrifice ratio of $2 \times 2 = 4$ is consistent with Ball (1994). The coefficient on productivity is about -1.7. The supply shocks are not significant as one would expect with the CPI-X price index.

The last column shows the findings for the case when an alternative measure of unemployment, demographically adjusted unemployment series, is used. The series is calculated as the weighted average of unemployment weights of various age groups. As opposed to the usual unemployment rate, the weights are constant and are calculated as fractions of various age groups in the labor force in 1985. It is interesting to consider this series because some authors (Shimer, 1998) have suggested that demographic factors may be able to account for a substantial portion of the variation in the natural rate. I find that the results with this specification are very similar to the baseline in terms of the width of the NAIRU confidence intervals, significance of the Phillips curve slope and the magnitude of the sacrifice ratio.

The robustness checks in this section confirm that the productivity model improves upon the random walk model.

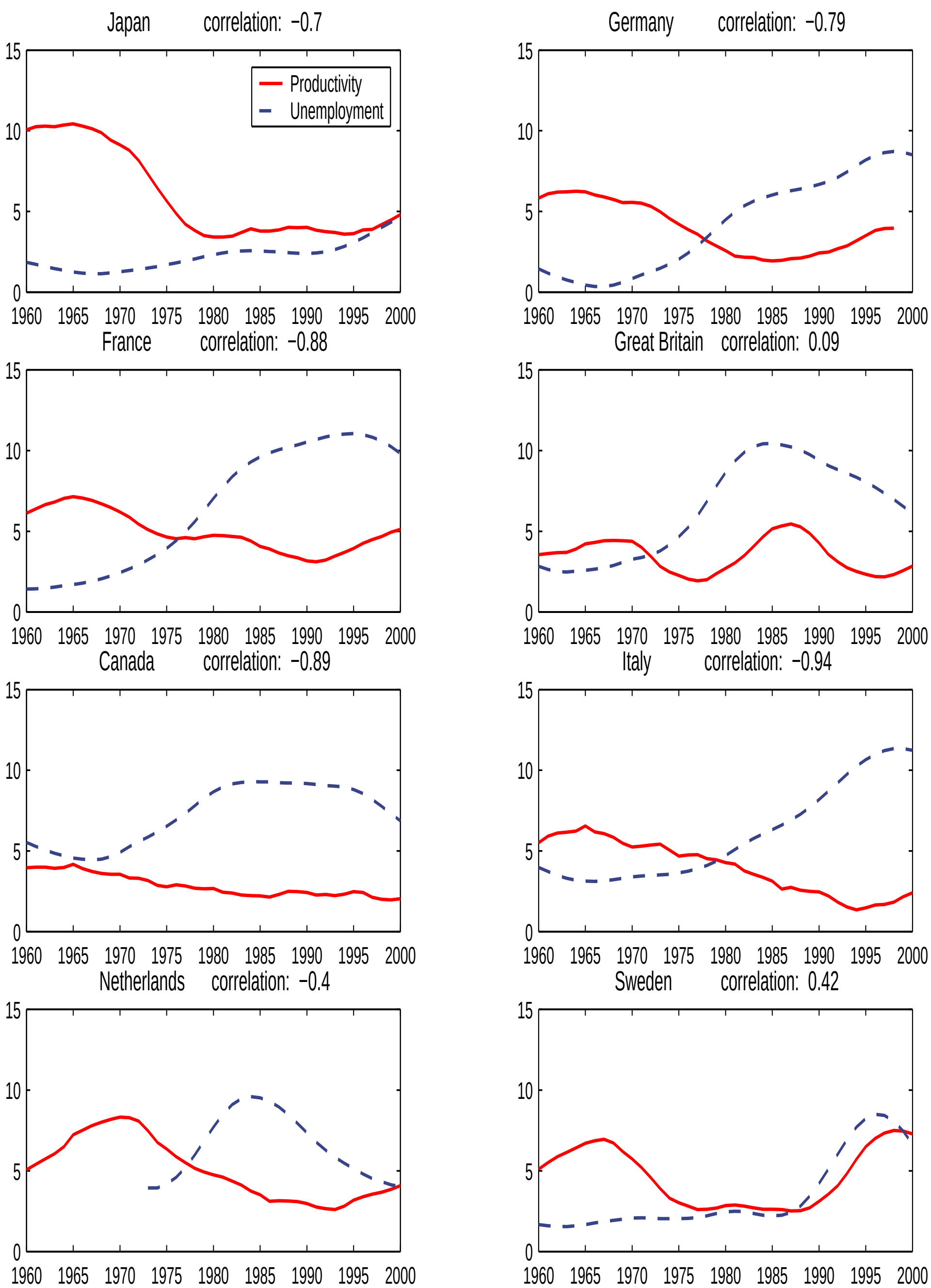
6. INTERNATIONAL EVIDENCE

Existing empirical work investigating the relationship between productivity growth and the natural rate focuses almost exclusively on the US data. One reason for this is the lack of comparable international productivity data. Fortunately, the shortage of higher frequency data is not such a serious problem with respect to the relationship between the long-run trends. In this case, the range of the data matters more than frequency and consequently 40 years of annual data are almost as valuable as 40 years of quarterly data.

Laubach (2001) illustrates other difficulties of estimating the Phillips curves with TV-NAIRUs for various countries. Laubach argues that the Phillips curves (2) produce NAIRU estimates that mimic the low frequency movements in unemployment rates only after a somewhat ad hoc adjustment. An alternative feasible approach with annual data, is to evaluate the relationship between unemployment and productivity trends. It is reassuring that the unemployment trends depicted in Figure 10 are broadly similar to Laubach's (2001) preferred estimates of the natural rates based on the Phillips curves.

Figure 10 shows the trends in unemployment and in the level of productivity growth and correlations between the two variables for eight non-

FIG. 10. International Trends in Productivity and Unemployment



Notes: The trends are estimated using the Baxter and King (1999) bandpass filter with upper cutoff frequencies of 15 years.

TABLE 6.

Correlations Between Productivity and the NAIRU in International Data

h	USA	Japan	Germany	France	Britain	Canada	Italy	Neth	Sweden
1	0.04	0.52	0.70	0.38	0.13	0.35	0.06	-0.29	0.72
2	0.12	0.45	0.64	0.33	0.17	0.45	-0.01	-0.15	0.76
3	0.06	0.39	0.70	0.38	0.19	0.49	0.05	-0.70	0.79
4	0.07	0.28	0.59	0.34	0.21	0.64	-0.20	-0.66	0.82
5	-0.06	0.38	0.65	0.39	0.32	0.54	-0.30	-0.79	0.84
6	0.01	0.46	0.60	0.43	0.34	0.64	0.02	-0.88	0.87
7	-0.12	0.31	0.52	0.51	0.39	0.64	0.12	-0.96	0.87
8	-0.23	0.36	0.52	0.52	0.39	0.44	-0.03	-0.89	0.89
9	-0.32	0.37	0.59	0.64	0.52	0.39	-0.03	-0.87	0.90
10	-0.49	0.45	0.64	0.70	0.54	0.50	-0.42	-0.86	0.90
Mean Diff	-0.09	0.40	0.61	0.46	0.32	0.51	-0.07	-0.70	0.84
Level	-0.81	-0.70	-0.79	-0.88	0.09	-0.89	-0.94	-0.40	0.42

Notes: The correlations are calculated from annual data, 1960–2002.

US countries: Japan, Germany, France, Great Britain, Canada, Italy, the Netherlands and Sweden. In most cases there are sizeable negative correlations between the *level* of productivity growth and the natural rate of unemployment estimated by the long-run trend. The average correlation between the level of productivity growth and the NAIRU is -0.54. Two countries that do not exhibit large negative correlations are Great Britain and Sweden.

Table 6 displays the correlations between the unemployment trends and changes in productivity growths $\theta^*_{t+h} - \theta^*_t$ for various horizons h. There is more evidence for a negative relationship between the level of productivity growth and the natural rate than between the change in productivity growth and the natural rate. This finding is robust across most countries and horizons h. In all countries except for the Netherlands the correlations between the NAIRU and the change in productivity growth are either ambiguous or, more often, positive and large. The last line of Table 6 shows the correlations between the levels of productivity growth and the natural rate. These correlations mimic the findings for the US in that they are negative in most cases and often large, Great Britain and Sweden are the two exceptions.

Overall, the international data support the evidence from the US on the relationship between the productivity and the natural rates. For most countries there is a strong negative correlation between the level of productivity growth and the natural rate. In contrast, the data speak less clearly about the sign of the correlation between the change in productivity growth and the NAIRU.

7. CONCLUSION

This paper shows that the estimate of the natural rate can be improved considerably by using information contained in the trend of productivity growth. The proposed econometric model provides a more precise estimate and a more realistic time profile of the NAIRU. Both these results are prerequisites for superior estimates of the unemployment gap. Policy makers often consider this gap when making interest rate decisions.

I also find support for a negative correlation between the natural rate and the level of productivity growth both in the US and international data. This seems to contradict many theoretical models proposed to explain the recent decline in the natural rate. However, the theory and the empirics are reconciled if workers update their estimates of the trend in productivity growth very slowly. Nevertheless, explaining the negative correlation between the natural rate and the level of productivity growth is an important area of future research.

APPENDIX: DATA DESCRIPTION

This appendix describes the data used in the paper. The US data are quarterly, 1960:1–2002:1. They are obtained from the DRI database. In the baseline model, inflation is constructed from the CPI for all urban consumers (PUNEW in the DRI mnemonics). Unemployment is the unemployment rate for all workers of 16 years and over (LHUR). Productivity is the output per hour in non-farm business sector for all persons (LBOUTU). Supply shocks are calculated following Staiger *et al.* (1997). Define the price index for food and energy as $p_{fe} = 0.66 \cdot p_f + 0.34 \cdot p_e$, where p_f is the "producer price index of foodstuffs and feedstuffs" (PW1100) and p_e is the "producer price index of crude fuel" (PW1300). Supply shocks are constructed as the demeaned difference between the inflation of p_{fe} and CPI inflation.

Alternative series in the Robustness section 5 are measured as follows. Productivity in manufacturing is the LOUTM series. GDP implicit deflator inflation is measured by GDPD96. CPI-X inflation is measured by the CPI U index less food and energy, PUXX. Finally, unemployment for men of 25–54 years is LHMU25.

International data are annual, 1960–2001. They are downloaded from the Bureau of Labor Statistics web site. The productivity data are output per hour in manufacturing data from `http://www.bls.gov/news.release/prod4.t01.htm`. The unemployment data are the civilian unemployment rates approximating US concepts from Table 2 of Comparative Civilian Labor Force Statistics available at `ftp://ftp.bls.gov/pub/special.requests/ForeignLabor/flslforc.txt`.

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