Productivity and the Natural Rate of Unemployment*

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I propose an econometric model that improves on existing methods for estimating the natural rate of unemployment (NAIRU) by using information contained in the trend of productivity growth. My approach outperforms existing methods in several respects. First, the method makes it possible to estimate the natural rate more precisely. The width of the 95% confidence intervals shrinks from over 4 to 3 percentage points on average. Second, the productivity-augmented model generates a more realistic time profile of the NAIRU. Third, the new model implies more plausible estimates of the Phillips curve slope and of the sacrifice ratio. Finally, the new estimate of the natural rate performs better in an out-of-sample inflation forecasting exercise. I also test whether the natural rate is correlated with the level or change of the productivity growth trend. I find support for the "level" hypothesis in both the US and international data.

Keywords: natural rate of unemployment, productivity, Phillips curve, time-varying parameters, Kalman filter

JEL Classification: C22, E31, E50

1. INTRODUCTION

Two salient features of the natural rate of unemployment (NAIRU) are its substantial time variation and the considerable uncertainty that surrounds it. Furthermore, recent empirical work (Staiger, Stock and Watson (2001)) has found a strong negative correlation between the natural rate and the trend of productivity growth in the United States. This paper proposes an econometric model that improves on existing methods for estimating the NAIRU by using information contained in the trend of productivity growth.

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The method makes it possible to estimate the natural rate more precisely and outperforms existing approaches in several other respects.

Many authors (Gordon (1997), Gordon (1998), Katz and Krueger (1998), Staiger *et al.* (2001), and others) document that the time profile of the natural rate varies substantially over time. For example, Gordon (1997)'s preferred estimate of the NAIRU declines from a peak of about 6.5% in 1980 to a low of 5.6% by mid-1996. Besides being of interest for the monetary authority, the estimate of the natural rate is crucial for producing accurate inflation forecasts. The failure to account for the time variation in the natural rate caused the forecasting performance of the standard Phillips curves to break down in the late 1990's (Ball and Moffitt (2001)). Consequently, it is not acceptable to model the natural rate as a constant.

Staiger *et al.* (2001) report that the trends of unemployment and productivity growth co-move strongly. I reproduce their finding in Figure 1. The correlation between unemployment and productivity growth trends over the period 1960-2001 is -0.8. Descriptive statistics for productivity and unemployment displayed in Table 1 also illustrate this inverse relationship. Productivity growth was rapid before 1973, slowed down in the after 1973 for more than twenty years and then resumed vigorously after 1995. The average unemployment rate on the other hand was more than 1 point higher between 1973 and 1995 than before and after that period. This is an impressive result since no unemployment data are used to construct the productivity data.

The existing research (Staiger, Stock and Watson (1997)) has modelled the time variation in the natural rate of unemployment as a function of time, spline, and alternatively as an unobserved random walk. The spline approach faces two criticisms. The model is not very flexible and modelling the natural rate as a time polynomial is rather ad hoc.

This paper extends the random walk framework by using information contained in the trend of productivity growth. The original random walk formulation assumes that the natural rate is completely driven by some unobserved white noise variable. Adding the productivity growth trend to the model explains a large part of variation in the NAIRU and significantly shrinks the unobserved part. Intuitively, including a relevant variable in the regression improves its quality.

My approach outperforms existing methods in several respects. First, the method makes it possible to estimate the natural rate more precisely. The width of the 95% confidence intervals shrinks from over 4 to 3 percentage points on average. Second, the productivity-augmented model generates a more realistic time profile of the NAIRU. Third, the new model implies more plausible estimates of the Phillips curve slope and of the sacrifice ratio. Fourth, the new estimate of the natural rate performs better in an out-of-sample inflation forecasting exercise. Finally, the new productivity-augmented model is more robust than the existing random walk model to the choice of the signal-to-noise ratio, a parameter that has to be imposed rather than estimated.

I also test whether the natural rate is correlated with the level or change of the productivity growth trend. I find support for the “level” hypothesis in both the US and international data.

This is surprising because many models proposed to explain the relationship between the natural rate and productivity growth (Brown (1984), Ball and Moffitt (2001), Mankiw and Reis (2003)) are consistent with the “change” hypothesis. The starting point of these models is that workers’ estimates of productivity growth adjust slowly to true productivity growth. Consequently, after an increase in productivity growth workers temporarily demand wages below their marginal product. However, after some time workers’ wage targets adjust to their marginal product. As a result, these models explain the negative correlation between the natural rate and changes in productivity growth, rather than the natural rate and the level of productivity growth. Interestingly, there is not much theoretical work explaining negative correlation between the natural rate and the level of productivity growth.

The paper is organized as follows. Section 2 reviews the theoretical literature on the relationship between the natural rate and productivity. Section 3 proposes the econometric model and discusses the econometric issues. Section 4 reports the empirical findings of the baseline model for the US. Section 5 summarizes the robustness results and tests the “level vs. change” hypothesis. Section 6 focuses on the international evidence on the relationship between the productivity growth and the natural rate. Section 7 concludes.

2. PRODUCTIVITY AND THE NAIRU: THEORY REVIEW

There exist several explanations for the inverse relationship between productivity growth and the natural rate of unemployment. While the models are based on different assumptions about the causes of the mismatch between the productivity perceived by workers and firms, their implications are similar.

Braun (1984) and Meyer (2001) assume that workers base their wage claims on a *real-time* estimate of the productivity trend. This estimate does not immediately completely respond to a productivity acceleration. In contrast, price-setting depends on the true productivity trend. Consequently, the productivity acceleration has an asymmetric effect on wages and prices because workers realize that the rate of productivity growth increased only with a lag. This results in a temporary fall in the natural rate of unemployment after the productivity growth speeds up.

Ball and Moffitt (2001) assume that the workers' real wage targets depend on both productivity and *aspirations.* Aspirations are a weighted average of the past real wages. Ball and Moffitt define the NAIRU as the rate of unemployment consistent with stable inflation and $\theta - A = 0$, where θ is labor-productivity growth and A is growth rate of wage aspirations. Ball and Moffitt "treat movement in $\theta - A$ as 'supply shocks' that shift the unemployment–inflation tradeoff for a given NAIRU" (p. 9). Once productivity growth speeds up, $\theta - A$ rises because aspirations respond sluggishly, and consequently unemployment can fall below the natural rate temporarily without accelerating inflation.

Mankiw and Reis (2003) propose a *sticky-information* model in which each period a randomly chosen fraction of workers updates information on productivity. The rest of workers, due to costly information gathering, uses outdated information. Consequently, the past expectations of the current productivity growth enter the Phillips curve. As a result unexpected increase in productivity growth temporarily lowers inflation and the natural rate.

Blanchard and Katz (1997) review theoretical explanations of the relationship between productivity growth and the natural rate. Higher productivity growth often comes with a structural change. In that case old jobs are destroyed and replaced by new ones. As a result, productivity acceleration is likely to raise the structural unemployment and possibly the natural rate. While this hypothesis implies that unemployment and productivity are correlated, it gets the sign wrong.

However, in the literature on job search there actually are theoretical models which imply negative correlation between unemployment and productivity growth. For example, Mortensen and Pissarides (1998) argue that the sign of correlation between unemployment and productivity depends on the costs of implementing new technology. When these renovation costs are small enough, higher productivity induces lower unemployment because firms update their technology continually and create jobs. Consequently, this strand of research can possibly explain the negative correlation between productivity growth and the NAIRU. However, it is an open question what fraction of productivity growth actually requires structural adjustment and what fraction happens without it.[1]

3. ECONOMETRIC MODEL

The NAIRU or "the non-accelerating inflation rate of unemployment" is typically estimated in the Phillips curve framework as the rate of unemployment that is consistent with stable inflation expectations. This section first reviews existing methods of modelling the natural rate both as a constant and in the time-varying parameter framework. I then propose the productivity-augmented model and discuss some econometric issues.

Assume for the moment that the natural rate $\bar{u}$ is constant. To estimate the NAIRU, start with the expectations-augmented Phillips curve,

$$\Delta\pi_t = \gamma(L)(u_{t-1} - \bar{u}) + \delta(L)\Delta\pi_{t-1} + \alpha(L)X_t + \varepsilon_t, \tag{1}$$

where $\gamma(L), \delta(L)$ and $\alpha(L)$ are lag polynomials and X_t includes the supply shocks. Phillips curve (1) assumes that inflation expectations follow random walk, $\pi_t^e = \pi_{t-1}$. The natural rate can be estimated by ordinary least squares (OLS) as the horizontal intercept. Specifically, after running the regression

$$\Delta\pi_t = \gamma_0 + \gamma(L)u_{t-1} + \delta(L)\Delta\pi_{t-1} + \alpha(L)X_t + \varepsilon_t$$

the estimate of the NAIRU is $\bar{u} = -\gamma_0/\gamma(1)$, where $\gamma(1)$ is the sum of unemployment coefficients.

The constancy of the natural rate is a very restrictive assumption. As Gordon (1997, p. 12) puts it, "the NAIRU is not carved in stone." Friedman (1968) defines the natural rate as the "level which would be ground out by the Walrasian system of general equilibrium equations, provided there is imbedded in them the actual structural characteristics of the labor and commodity markets." To capture the effects of changes in these characteristics on the NAIRU, Staiger, Stock and Watson (1997) propose the unobserved random walk (or time-varying parameter) model,

$$\begin{aligned} \Delta\pi_t &= \gamma(L)(u_{t-1} - \bar{u}_{t-1}) + \delta(L)\Delta\pi_{t-1} + \alpha(L)X_t + \varepsilon_t, \\ \bar{u}_t &= \bar{u}_{t-1} + \eta_t, \qquad \mathrm{var}(\eta_t) = \lambda \mathrm{var}(\varepsilon_t). \end{aligned} \tag{2}$$

[1] Mortensen and Pissarides (1998) give the following examples. Switching from typewriters to word processors requires the firms to train the staffs but by and large the same employees carry on working together. In contrast, Mortensen and Pissarides, p. 735, ague that structural adjustment occurred when mechanization came to the textile industry because "...the preexisting organization of production based on the cottage industry could not carry on. ... [C]ottage industry jobs were destroyed and new ones, now housed in factories, took their place."

The natural rate $\bar{u}_t$ is now assumed to follow the random walk. The variation in $\bar{u}_t$ is governed by the signal-to-noise parameter $\lambda \equiv \text{var}(\eta_t)/\text{var}(\varepsilon_t)$. If $\lambda = 0$, the NAIRU is constant and (2) reduces to (1) above. The model (2) is estimated by the maximum likelihood (ML) using the Kalman filter algorithm.

The random walk model is a flexible device that captures the unobserved time-variation in the natural rate. However, when there are variables that are informative about the NAIRU, it is more efficient to include them in the model. This decreases the unexplained $\text{var}(\eta_t)$ and raises the fraction of $\bar{u}_t$ explicitly modelled by observables.

It is easy to generalize the Kalman filter framework by including exogenous variables Z_t in the second (state-evolution) equation of (2),

$$\begin{aligned} \Delta\pi_t &= \gamma(L)(u_{t-1} - \bar{u}_{t-1}) + \delta(L)\Delta\pi_{t-1} + \alpha(L)X_t + \varepsilon_t, \\ \bar{u}_t &= \bar{u}_{t-1} + \beta^\top \Delta Z_t + \eta_t, \qquad \text{var}(\eta_t) = \lambda \text{var}(\varepsilon_t). \end{aligned} \tag{3}$$

In the model (3) a fraction of the variation in the state variable $\bar{u}_t$ is explained by exogenous variables in Z_t. Consequently, the variance of the error term η_t in the random walk model (2) is greater that the variance of the error in (3) and as a result model (3) explains $\bar{u}_t$ better.

The natural rate in (3) is modelled as a random walk driven by the exogenous variables Z_t and the error term η_t. This specification is chosen, as is usual in the literature, instead of the white noise specification $\bar{u}_t = \beta Z_t + \eta_t$, to allow for persistent deviations of $\bar{u}_t$ from βZ_t. It is important to note that the specification (3) implies that differences in Z_t affect differences in the natural rate $\bar{u}_t$, or equivalently that levels of Z_t affect levels of $\bar{u}_t$.

In the baseline specification the exogenous variables Z_t consist of the productivity trend θ_t^*, obtained by the Kalman filter as explained below. Specification (3) assumes that that Z_t only influences $\bar{u}_t$. In particular, there is no direct effect of Z_t on $\Delta\pi_t$ without affecting the natural rate. This assumption is justified since the productivity trend θ_t^* varies slowly. The supply shocks X_t are, in contrast, extremely volatile and therefore I follow existing literature in assuming that they only affect $\Delta\pi_t$, not the natural rate $\bar{u}_t$.

Econometric Issues

The productivity trend θ_t^* is estimated by the random walk plus noise (or local level) model,

$$\theta_t = \theta_t^* + z_{Tt}, \qquad \theta_t^* = \theta_{t-1}^* + z_{Pt}, \qquad \text{var}(z_{Tt}) = \lambda_\theta \text{var}(z_{Pt}) \tag{4}$$

where θ_t is the observed, measured productivity growth data, θ_t^* is the unobserved trend to be estimated and z_{Tt} and z_{Pt} are the temporary and permanent shocks to productivity, respectively. This specification is a flexible device that makes it possible to extract the long-run trend from the time series using the Kalman filter algorithm. It is an alternative to the more common filters, such as the Hodrick–Prescott filter. The advantage of the Kalman filter model (4) is that the algorithm produces an optimal estimator of the trend (the minimum mean squared error linear estimator), see e.g. Harvey (1989). Another reason for the use of the detrending model (4) is that it fits into the Kalman filter framework employed in the paper.[2]

I assume that the disturbances ε_t and η_t in (2) and (3) are i.i.d. normal $\mathcal{N}(0, \mathrm{var}(\varepsilon_t))$ and $\mathcal{N}(0, \mathrm{var}(\eta_t))$, respectively. Furthermore, the disturbances ε_t and η_t are also assumed to be uncorrelated. I estimate the parameters $\{\gamma(L), \delta(L), \alpha(L), \beta, \mathrm{var}(\varepsilon_t)\}$ by the maximum likelihood, as described in Harvey (1989).

The amount of the time variation in $\bar{u}_t$ is governed by the signal-to-noise parameter λ. Since the NAIRU varies slowly over time, the variance of η_t is usually very small compared to the variance of ε_t. Consequently, the estimate of $\mathrm{var}(\eta_t)$ has bad small-sample properties—it is estimated very imprecisely, with a downward bias. Besides, in small samples the distribution of the signal-to-noise ratio λ has a non-zero probability at zero, a so-called pile-up problem. This results in the implied natural rate of unemployment being too smooth, often almost constant. Consequently, I follow existing literature (Staiger *et al.* (1997), King, Stock and Watson (1995) and others) in imposing a reasonable value for λ instead and estimating the remaining parameters by ML. Interestingly, the estimate of the natural rate in the productivity model (3) is considerably more robust to the choice of λ than in the random walk model, as documented in section 5.

Stock and Watson (1998) propose an alternative to imposing λ. The method consists of conducting the sup-Wald structural break test for a break in the constant in the Phillips curve. One then compares the test statistic to the table of Stock and Watson (1998) critical values and retrieves the implied median-unbiased estimate of λ together with its confidence intervals. I estimate the signal-to-noise ratios λ using this method and report the median-unbiased estimates of $\mathrm{var}(\eta_t)$ in the last line of Table 2 below. However, I do not use the method in the calculations below because the con-

[2] I consider the productivity trend θ_t^* obtained by the bandpass filter in section 5 below. Figure 2 compares the productivity trends measured by the Kalman and bandpass filters.

fidence intervals for λ tend to be very wide and the estimated signal-to-noise ratios are less satisfactory than the imposed ones in some cases.

4. EMPIRICAL RESULTS

In this section I compare three benchmark models for modelling the natural rate: the constant NAIRU model (1), the random walk model (2) of Staiger *et al.* (1997), and the productivity-augmented model (3). The major flaw of the first model is the assumption of the constant NAIRU. The random walk model does not do a good job in several respects: slope of the Phillips curve and the implied sacrifice ratio, NAIRU confidence intervals, and time profile of the natural rate. The productivity model alleviates these shortcomings.

Table 2 reports the main findings. Column one summarizes the traditional backward-looking Phillips curve with the constant NAIRU. This specification is estimated by OLS. Its principal strength is that the statistics are in line with conventional wisdom. The lags of inflation, unemployment, and supply shocks are significant. The value of the slope, $\gamma(1)$, is comparable to the findings of other authors. Finally, the implied sacrifice ratio, the unemployment cost of reducing inflation, is in the upper range of estimates obtained by Ball (1994) and others. In light of the recent decline of the natural rate, its assumed constancy is a crucial shortcoming. The reported estimate of the natural rate of about 6% can be in principle interpreted as the average value of the true time-varying NAIRU (TV-NAIRU). However, it is questionable how useful for the monetary authority it is to know the average natural rate when the NAIRU varies substantially. The confidence intervals are calculated following Staiger *et al.* (1997) using the Anderson–Rubin exact method based on inverting the F statistic of H_0: $\bar{u} = u_0$ for various values of u_0.

The second column of Table 2 displays the results of the random walk model (2). While this model no longer restricts the natural rate to be constant, it does quite badly in several other respects. First, as documented in Figure 3, the implied natural rate is too choppy to represent correctly our intuition. Second, the slope of this Phillips curve is smaller in magnitude than the slope of the OLS Phillips curve and as a result the implied sacrifice ratio is very high. Even worse, the slope of the Phillips curve is not statistically significant. The slope $\gamma(1)$ enters the denominator of the estimate of the NAIRU, which causes the natural rate to be unidentified when the slope is zero. Similarly, when $\gamma(1)$ is small the confidence intervals for the natural rate tend to be extremely wide (see Staiger *et al.* (1997)).[3]

Column 3 of Table 2 describes the implications of the productivity model. Adding the productivity trend results in a clear improvement of the performance of the model. The slope of the Phillips curve $\gamma(1)$ is considerably greater in magnitude than the slope in the random walk model. This results in sharp increase of precision of the NAIRU estimates. The width of the confidence intervals for the natural rate shrinks on average by about 25%. Finally, the sacrifice ratio is consistent with the conventional wisdom.

The productivity growth is borderline significant with the p value of 0.048. The sensitivity of the natural rate with respect to the productivity growth, β, is about -2, which means that if the level of productivity growth increases by 1%, the natural rate declines by 2%. Assuming the productivity growth went up by 0.6% in the late 1990s, this translates into a 1.2% fall in the NAIRU, as is also documented in Figure 5.

4.1. Confidence Intervals

Because the slope of the random walk Phillips curve is close to zero, the natural rate is hard to pin down and consequently its confidence intervals are wide. The productivity model, in contrast, implies a greater Phillips curve slope, which narrows the NAIRU confidence intervals. This subsection compares the confidence intervals for implied by various models.

Figure 4 compares the widths of the NAIRU confidence intervals implied by the random walk and the productivity models. The confidence intervals are calculated from the variance of the Kalman smoother estimate of $\bar{u}_t$ with a delta method correction for parameter uncertainty due to Ansley and Kohn (1986). The method is consistent with Staiger *et al.* (1997).

The width of confidence intervals shrinks from 3.1 to 4.1 percentage points on average, by about 25%, with the productivity model compared to the random walk model. The black solid line in Figure 4 depicts the replication with quarterly data, 1960–2002 of the 95% confidence intervals of Staiger *et al.* (1997). In fact, even though the point estimates of the natural rates in Figure 5 differ by up to 1%, the shaded confidence band for the productivity model is for most periods within the confidence band of the random walk model.

[3]The estimates of slopes of the OLS and random walk Phillips curves are consistent with other specifications in the literature, e.g. Staiger *et al.* (1997) and Staiger *et al.* (2001), respectively.

A major problem of the model (2) is that the time variation natural rate $\bar{u}_t$ is driven exclusively by the white noise η_t. This is a reasonable solution when one is agnostic about the possible causes for the movements of the NAIRU. However, when we have candidates that might plausibly be correlated with the NAIRU, it is beneficial to use the additional information contained in these series. If the correlation between these variables Z_t is strong enough, adding them to the econometric model increases the quality of the estimated natural rate and the parameters. Intuitively, including a relevant explanatory variable in the regression improves the precision of estimates.

4.2. Time Profile of the Estimates of the Natural Rate

One important shortcoming of the random walk model is that it implies unrealistic estimate of the time profile of the natural rate. There is not only evidence that the NAIRU is not constant, we actually have a prior on how it varies. We typically think of it as a slowly varying, smooth function of time. Large abrupt changes in the natural rate are very unlikely.

The NAIRU time profile of the random walk model is displayed in Figure 3, a replication of Staiger *et al.* (1997)'s Figure 6. There are at least two problems with the NAIRU profile: it is both excessively sensitive and excessively smooth. More precisely, there is too much of high-frequency variation and not enough low-frequency variation in the natural rate. The natural rate of Figure 3 is not very smooth, at the same time its constancy cannot be rejected. Unfortunately, increasing the λ parameter affects the high-frequency variation in the natural rate and does not improve the results much.[4] The random walk model substitutes the lack of low-frequency variation in the natural rate by the high-frequency variation. Figure 5 documents that this does not work satisfactorily. Both the rise in the NAIRU in the late 1970's and its fall in the late 1990's are much less pronounced for the random walk model than for the productivity model.

Interestingly, the shape of the time-varying NAIRU implied by the productivity model is much closer to the conventional wisdom. This is because the productivity growth adds more low-frequency variation and at the same time decreasing λ makes it possible to lower the high-frequency variation in the NAIRU.

One can decompose the variation of the natural rate by frequency using its spectrum. The spectra of processes with more low-frequency variation

[4]I explore the effects on the estimates of the natural rate of imposing other values of λ in subsection 5.2 below.

have more mass close to the origin. Figure 6 shows that the spectrum of the productivity natural rate has much more low-frequency variation than the random walk NAIRU.[5] This confirms the previous intuitive observations about the lack of low-frequency variation of the random walk estimates of the natural rate.

4.3. Slope of the Phillips Curve and Sacrifice Ratio

I note above that using the information from the productivity growth trend increases the magnitude of the Phillips curve coefficient and its significance. The intuition for this finding comes from a model of omitted variable bias in the OLS regression. If the productivity growth is a relevant omitted variable in the Phillips curve, it biases the estimate of the slope downward. This happens because the bias in the slope is proportional to the product of the slope and the correlation between productivity and unemployment. Since both the correlation and the slope are negative, the slope is biased upward, towards zero.

The magnitude of the slope of the Phillips curve determines the sacrifice ratio, the cost in percentage points of unemployment of decreasing inflation by one percentage point. Sacrifice ratio is estimated from the Phillips curve as the long-run response IR_{un} of inflation π_t to a one percentage point increase in the unemployment rate over one year. To get the intuition, suppose one has the Phillips curve with no inflation lags on the right-hand side. The the long-run response of inflation to a one percentage point increase in unemployment over one year period is the sum of the unemployment coefficients $\gamma(1)$ or equivalently an increase in unemployment by $|1/\gamma(1)|$ percentage points results in 1 percentage point decline in inflation rate.

Figure 7 compares the long-run responses of the level of inflation to a 1% shock to unemployment for the productivity and random walk models. As already suggested by the slopes of the Phillips curves, the long-run response of the productivity model is by about 30% bigger than that of the random walk model, -0.08 vs. -0.11. This translates to different sacrifice ratios, as documented by the last but one line of Table 2. The estimate of the sacrifice ratio implied by the random walk model is substantially higher than the estimates from the OLS and productivity models. Assuming the coefficient of 2 in Okun's law, the output cost of disinflation is about 6 for the random walk model and about 4.5 for the productivity model. Ball (1994)'s estimates

[5]The spectrum of the random walk is infinite at zero frequency. However, one can still use the sample estimate of the spectrum to decompose the variation of a stochastic process.

of sacrifice ratios for the disinflation episodes in the OECD countries range between 2 and 4. Consequently, the sacrifice ratio implied by the random walk model seems too high. In contrast, the sacrifice ratio implied by the productivity model is more in line with the conventional wisdom.

4.4. Forecasting

It is standard to use the Phillips curve as an inflation forecasting tool. To produce h-period ahead inflation forecasts the following modification of the Phillips curve (1) is often used,

$$\Delta_h \pi_t = \gamma(L)(u_{t-1} - \bar{u}_{t-1}) + \delta(L)\Delta\pi_{t-1} + \varepsilon_t, \tag{5}$$

where $\Delta_h \pi_t = \pi_{t+h} - \pi_t$ is the h-period change in inflation. Stock and Watson (1999) argue that the Phillips curve (5) generates more accurate one-year ahead inflation forecasts than the majority of other relationships.

To evaluate the quality of two alternative estimates of the natural rate, $\bar{u}_{t,1}$ and $\bar{u}_{t,2}$, I employ the following procedure. Given $\bar{u}_{t,i}$ and inflation and unemployment data I estimate the regression (5) and produce inflation forecasts both in out-of-sample and in-sample framework. The out-of-sample forecasts are generated by rolling regressions that are recursively estimated based on variables dated time $1, \ldots, t$. Because it is first necessary to use the information in the whole sample $1, \ldots, T$ to estimate the NAIRU, $\bar{u}_t$, these regressions should not be interpreted as a real-time procedure. However, the procedure is still valid for evaluation of the quality of alternative NAIRU estimates.[6] As an alternative to the out-of-sample procedure one can produce the forecasts in an in-sample framework as fitted values from regression (5) based on the information $1, \ldots, T$.

Table 3 displays the mean squared errors (MSE) of the forecasts of the productivity and random walk models relative to the MSE of the constant NAIRU for various forecasting horizons h. The out-of-sample forecasts of the productivity model are on average by 9% better than the constant NAIRU forecasts and by 5% more precise than the random walk forecasts. The differences are more pronounced at longer forecasting horizons. This is because the slope of the Phillips curves for longer horizons h is greater. This in turn is intuitive, since when the unemployment is above the NAIRU, one would expect inflation steadily increasing. As a result, $\Delta_h \pi_t \approx h \times \Delta_1 \pi_t$. The right panel of Table 3 displays the in-sample results. The differences in

[6]One can in principle imagine implementing this procedure in real-time-like framework and estimating the models (2) or (3) at each time period t. However, because there is a lot of uncertainty about the natural rate at the end of the sample, this would probably produce extremely noisy inflation forecasts and is not pursued here.

quality of various models are not as significant as in the out-of-sample case, however, the productivity model still performs best and the constant model does relatively poorly.

To sum up, accounting for the time-variation in the natural rate results in more precise inflation forecasts. These forecasts are further improved by using the information in the productivity growth.

5. SPECIFICATION TESTING AND ROBUSTNESS

This section considers various issues in specification testing. I first test whether the natural rate is correlated with *the levels or the changes in productivity growth*. Then I focus on the choice of the signal-to-noise ratio λ. Finally, I investigate whether my findings from previous sections hold for alternative inflation expectations, productivity, unemployment and inflation series.

5.1. Differences or Levels?

Above I investigate the relationship between the levels of the NAIRU and the productivity growth. As discussed in section 2, many theoretical models imply, however, the correlation between the level of natural rate and the change in productivity growth. I now focus on this relationship.

Informally, the last row of Table 1 suggests that the relationship between the change in productivity growth and the NAIRU is empirically not as strong as between the level of productivity growth and the NAIRU. The average change in productivity growth was small during 1960–1973, larger in 1974–1995, and still larger after 1995. Unemployment on the other hand was low before 1973 and after 1995 and low between 1974 and 1995. The first column in Table 6 below displays the correlations between productivity and unemployment trends in the US. The correlations between the changes in productivity growth $\theta^*_{t+h}-\theta^*_t$ and unemployment trend are often positive and tend to be negative only for very long horizons, for $h = 7$ years and longer. The correlation between the levels of productivity trend and the NAIRU, in contrast, is high and negative, -0.81. Finally, Figure 8 displays the trends in unemployment, productivity, and productivity growth, standardized to have zero mean and unit variance. The Figure confirms that the correlation between the change in productivity growth and the natural rate has a wrong sign. In particular, in the 1970's unemployment was rising, productivity growth was falling, yet the change in productivity growth was increasing.

To obtain more rigorous evidence I estimate model (3) with the change in productivity trend as an exogenous variable, $Z_t = \Delta\theta^*_t$. The first column of Table 4 summarizes this case. This model does not improve the random walk model. While the coefficient on the productivity variable $\Delta\theta_t^*$, it is insignificant. The confidence intervals for the natural rate are almost as wide as with the random walk model and the sacrifice ratio is very high.

The second column of Table 4 shows the findings for the model with the exogenous variable consisting of both productivity change and level, $Z_t = (\Delta\theta_t^*, \theta_t^*)^\top$. The change in productivity growth is insignificant. Other than that the implications of this model are similar to those of the baseline productivity model in Table 2. The size of the coefficient on productivity level, θ_t^*, is -1.9, the slope of the Phillips curve is greater than in the first column and the sacrifice ratio smaller. [7]

On the whole, both simple correlations and more rigorous Kalman filter model (3) support the "level" rather than "change" hypothesis.

5.2. Signal-to-Noise Ratios

In the previous computations I follow much of the literature in imposing the signal-to-noise ratio λ, as opposed to estimating it. The size of λ determines the high-frequency variation in the natural rate. The ideal signal-to-noise ratio is big enough for the implied natural rate to capture the time variation and at the same time small enough for the NAIRU to be smooth. I now investigate the sensitivity of the NAIRU time profiles to the choice of the signal-to-noise ratio.

Figure 9 compares the estimates of the natural rates for various λs for the random walk. The random walk model is more sensitive to the choice of λ. Unfortunately, none of the λs delivers the shape generated by the productivity model. The problem is that the choice of λ affects the high-frequency variation rather than the low-frequency variation in $\bar{u}_t$. Consequently, small values of the signal-to-noise ratio imply a smooth but almost constant estimate of the NAIRU in the random walk model. In contrast, large λ generates a volatile natural rate which fails to capture the smoothness.

Figure 10 displays the effect of changes in λ for the productivity model. Because the productivity variable soaks up much of the time-variation in the NAIRU, the results are robust to the choice of λ. The estimates of the natural rate look very similar for quite different values of λ. In fact, for any value of λ in the relatively wide 90% confidence interval of Stock and Watson (1998), the NAIRUs are very close to each other. This is yet another reassuring finding for the productivity model.

[7] One reason why the change in productivity growth $\Delta\theta_t^*$ does not perform well is that it is relatively volatile. However, the results remain to hold even after filtering $\Delta\theta_t^*$.

5.3. Alternative Time Series

This subsection discusses the implications of the above models with alternative productivity, unemployment, inflation and inflation expectations series. A broad conclusion is that, in most cases, the results reported in section 4 continue to hold.

The first column of Table 5 and and the first panel of Figure 11 summarize the findings for an alternative inflation expectations series. The inflation expectations were generated as inflation forecasts from an AR(4) process in $\Delta\pi_t$. Interestingly, this model performs even better than the baseline model. Both the Phillips curve slope and the productivity variable are significant. The mean width of confidence intervals for the natural rate shrinks to 2.7%, and the implied sacrifice ratio in terms of GDP is $2 \times 1.3 = 2.6$.

The second column reports the results for an alternative measure of productivity trend, the bandpass filter (see also Figure 2). Unfortunately, the coefficient on productivity, β, is not significant. However, the confidence intervals shrink considerably, to 2.8% and the sacrifice ratio is $2 \times 1.9 = 3.8$.

The third column describes the implications of model (3) with productivity measured as productivity in manufacturing, instead of the non-farm business sector productivity. Productivity in manufacturing is not a preferred measure of productivity because manufacturing is a relatively small fraction of the economy. It turns out that the correlation between this productivity measure and the NAIRU is not as high as in the case of non-farm business sector productivity. Consequently, the model does not do as well in shrinking the NAIRU confidence intervals and time profile as the baseline model. However, it does reduce the sacrifice ratio and increase the magnitude of the Phillips curve slope.

The fourth column collects the findings for the GDP deflator as a measure of inflation. These results mimic the implications of the baseline model. The slope of the Phillips curve is significant and the NAIRU confidence intervals are narrow. The sacrifice ratio is somewhat large, $2 \times 2.5 = 5$, however, it is still considerably lower than the random walk sacrifice ratio (not reported).

Inflation in the next column is measured by the CPI ex food and energy index. The findings are again similar to the baseline model. The natural rate confidence intervals are narrow, 2.8%. The sacrifice ratio is quite high, $2 \times 2 = 4$. The coefficient on productivity is about -1.7. As a reality check, the supply shocks are not significant, which is what one would expect with the CPI-X price index.

The last column shows the findings for the case when an alternative measure of unemployment is used, unemployment of men, 25–54 years of age. The productivity model (3) does not do a good job at explaining this natural rate of unemployment. At the same time, however, it does not do any worse than the random walk model (2).

The robustness checks in this section confirm that by and large the productivity model outperforms the random walk model.

6. INTERNATIONAL EVIDENCE

Existing empirical work investigating the relationship between the productivity growth and the natural rate focuses almost exclusively on the US data. One reason for this is the lack of comparable international productivity data. As a result, I do not investigate the relationship between productivity and the natural rate in the Kalman filter approach (3) described above. However, the lack of higher frequency data is not such a serious problem if researchers are interested in the relationship between the long-run trends. In this case, the range of the data matters more than frequency and consequently 40 years of annual data are almost as valuable as 40 years of quarterly data.

Laubach (2001) illustrates the difficulties of estimating the Phillips curves with TV-NAIRUs for international countries. Laubach argues that the Phillips curves (2) produce NAIRU estimates that mimic the low frequency movements in unemployment rates only after a somewhat ad hoc adjustment. An alternative feasible approach with annual data, is to evaluate the relationship between unemployment and productivity trends. It is reassuring that the unemployment trends depicted in Figure 12 are broadly similar with Laubach (2001)'s preferred estimates of the natural rates based on the Phillips curves.

Figure 12 shows the trends in unemployment and level of productivity growth and correlations between the two variables for eight non-US countries: Japan, Germany, France, Great Britain, Canada, Italy, the Netherlands and Sweden. In most cases there are sizeable negative correlations between the *level* of productivity growth and the natural rate of unemployment estimated by the long-run trend. The average correlation between the level of productivity growth and the NAIRU is -0.54. Two countries that do not exhibit a large negative correlations are Great Britain and Sweden.

Table 6 displays the correlations between the unemployment trends and changes in productivity growths $\theta^*_{t+h} - \theta^*_t$ for various horizons h. *There is more evidence for the negative relationship between the level of productivity growth and the natural rate than between the change in productivity growth and the natural rate.* This finding is robust across most countries and horizons h. The NAIRU and change in productivity growth are robustly negatively correlated only in case of the Netherlands. In all other countries

countries the correlations are either ambiguous or, more likely, positive and often large. The last line of Table 6 shows the correlations between the levels of productivity growth and the natural rate. These correlations mimic the findings for the US: they are in most cases negative and often quite sizeable, Great Britain and Sweden are the two exceptions.

To sum up, the international data support the evidence from the US on the relationship between the productivity and the natural rates. For most countries there is a strong negative correlation between the level productivity growth and the natural rate. In contrast, the data speak less clearly about the sign of the correlation between the change in productivity growth and the NAIRU.

7. CONCLUSION

This paper shows that the estimate of the natural rate can be improved considerably by using information contained in the trend of productivity growth. The proposed econometric model provides more precise estimate and time profile of the NAIRU. In addition, the Phillips curve slopes and sacrifice ratios implied by the new approach are more in line with conventional wisdom than those from the existing methods. I also find support for the negative correlation between the natural rate and the level of productivity growth both in the US and international data. This is intriguing because many theoretical model proposed to explain the recent decline in the natural rate imply the relationship between the NAIRU and the change in productivity growth. Explaining the negative correlation between the natural rate and the level of productivity growth is an important area of future research.

APPENDIX: DATA DESCRIPTION

This appendix describes the data used in the paper. The US data are quarterly, 1960:1–2002:1. They are obtained from the DRI database. In the baseline model, inflation is constructed from the CPI for all urban consumers (PUNEW in the DRI mnemonics). Unemployment is unemployment rate for all workers of 16 years and over (LHUR). Productivity is the the output per hour in non-farm business sector for all persons (LBOUTU). Supply shocks are calculated following Staiger *et al.* (1997). Define the price index for food and energy as $p_{fe} = 0.66 \cdot p_f + 0.34 \cdot p_e$, where p_f is the "producer price index of foodstuffs and feedstuffs" (PW1100) and p_e is the "producer price index of crude fuel" (PW1300). Supply shocks are constructed as the demeaned difference between the inflation of p_{fe} and CPI inflation.

Alternative series in the Robustness section 5 are measured as follows. Productivity in manufacturing is LOUTM series. GDP implicit deflator inflation is measured by GDPD96. CPI-X inflation is measured by CPI U index less food and energy, PUXX. Finally, unemployment for men of 25–54 years is LHMU25.

International data are annual, 1960–2001. They are downloaded from the Bureau of Labor Statistics web site. The productivity data are output per hour in manufacturing data from `http://www.bls.gov/news.release/prod4.t01.htm`. The unemployment data are the civilian unemployment rates approximating US concepts from Table 2 of Comparative Civilian Labor Force Statistics available at `ftp://ftp.bls.gov/pub/special.requests/ForeignLabor/flslforc.txt`.

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TABLE 1.

Averages for Productivity, Unemployment and Inflation

	1960-1973	**1974-1995**	**1995-2002**
Productivity Nonfarm Business	2.759	2.009	2.286
Unemployment	4.953	5.925	4.869
Inflation	2.818	4.234	2.378
Diff Productivity Nonfarm Business	-0.016	-0.001	0.026

Notes: Quarterly Data. The productivity means are calculated from the productivity trend generated by the Baxter and King (1999) bandpass filter with upper cutoff frequency of 60 quarters.

TABLE 2.

Estimation Results, Baseline Models

	OLS	**Random Walk**	**Productivity**
Sum of Coeffs on Unemployment	-0.199	-0.147	-0.212
Std Error on Sum of Unemployment	0.076	0.111	0.117
P value on Lags of Unemployment	0.009	0.000	0.000
P value on Lags of Inflation	0.000	0.000	0.000
P value on Supply Shocks	0.004	0.285	0.027
P value on Productivity	NaN	NaN	0.048
Coefficient on Productivity	NaN	NaN	-1.957
Mean Width of Confidence Intervals	3.078	4.114	3.091
Sacrifice Ratio	2.297	2.979	2.231
Estimate of the Signal-to-Noise Ratio	NaN	0.011	0.006

Notes: All p values are based on the White heteroscedasticity-robust standard errors. P value of 0 means less than 5×10^{-4}.

TABLE 3.

Out-of-Sample and In-Sam-le Forecasts, MSEs Relative to the Constant NAIRU MSE

	Out-of-Sample		**In Sample**	
Horizon h (quarters)	**Prod**	**RW**	**Prod**	**RW**
1	0.991	1.101	0.975	0.926
2	0.915	0.928	1.043	1.098
3	0.918	0.948	0.978	0.994
4	0.876	0.921	0.958	0.996
8	0.857	0.942	0.894	0.951
12	0.876	0.934	0.924	0.955
Mean	0.906	0.962	0.962	0.986

Notes: The out-of-sample results are based on the rolling regressions with increasing window and fixed initial date, 1960–2002.

TABLE 4.

Estimation Results, Difference vs. Level of Productivity

	Diff Model	**Level and Diff Model**	
Sum of Coeffs on Unemployment	-0.169	-0.202	
Std Error on Sum of Unemployment	0.101	0.115	
P value on Lags of Unemployment	0.000	0.000	
P value on Lags of Inflation	0.000	0.000	
P value on Supply Shocks	0.021	0.030	
P value on Productivity	0.441	0.064	0.6075
Coefficient on Productivity	-31.529	-1.876	-18.754
Mean Width of Confidence Intervals	3.866	–	
Sacrifice Ratio	2.686	2.349	
Estimate of the Signal-to-Noise Ratio	0.030	0.000	

Notes: All p values are based on the White heteroscedasticity-robust standard errors. P value of 0 means less than 5×10^{-4}.

TABLE 5.

MLE Estimation Results, Alternative Time Series

	ARExp	**Bps**	**GDPD**	**CPIX**	**Mnf**	**UM 25–54**
Sum of Coeffs on Unemployment	-0.277	-0.250	-0.212	-0.201	-0.227	-0.130
Std Error on Sum of Unemployment	0.118	0.131	0.097	0.087	0.133	0.108
P value on Lags of Unemployment	0.000	0.000	0.000	0.000	0.000	0.000
P value on Lags of Inflation	0.000	0.000	0.000	0.000	0.000	0.000
P value on Supply Shocks	0.018	0.027	0.021	0.017	0.230	0.009
P value on Productivity	0.033	0.133	0.114	0.039	0.074	0.120
Coefficient on Productivity	-1.821	-1.159	-2.384	-1.582	-1.688	-2.283
Mean Width of Confidence Intervals	2.699	2.808	3.155	2.423	2.846	4.729
Sacrifice Ratio	1.325	1.882	2.341	2.437	2.007	3.630
Estimate of the Signal-to-Noise Ratio	0.000	0.004	0.004	0.000	0.004	0.017

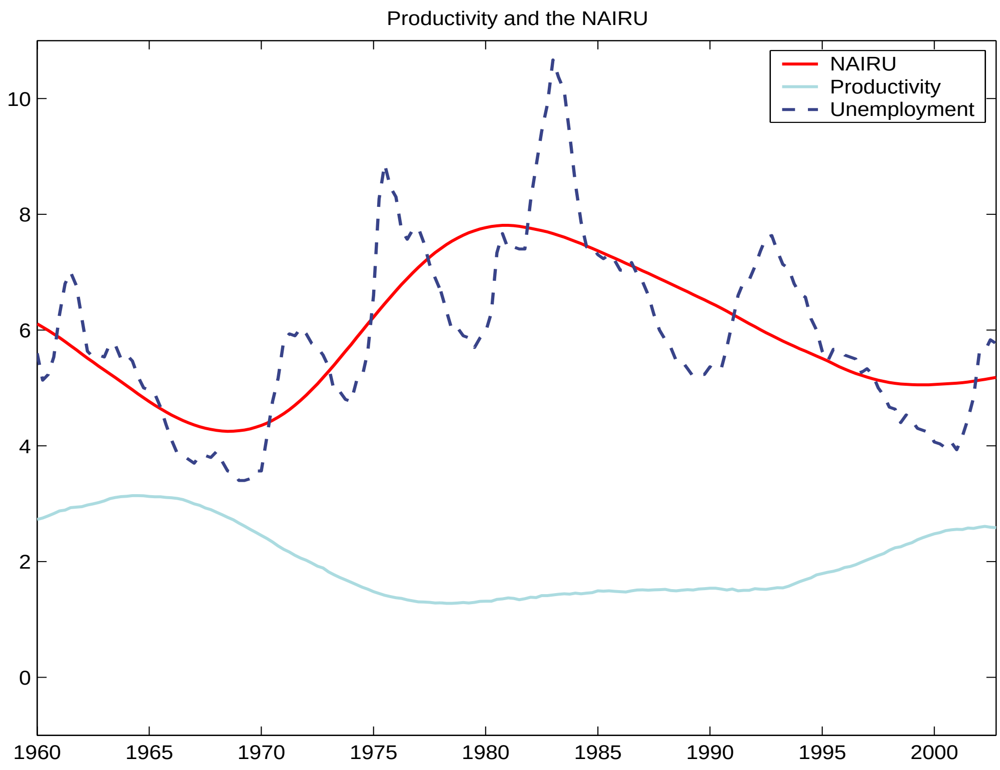
Notes: All p values are based on the White heteroscedasticity-robust standard errors. P value of 0 means less than 5×10^{-4}.

TABLE 6.

Correlations Between Productivity and the NAIRU in International Data

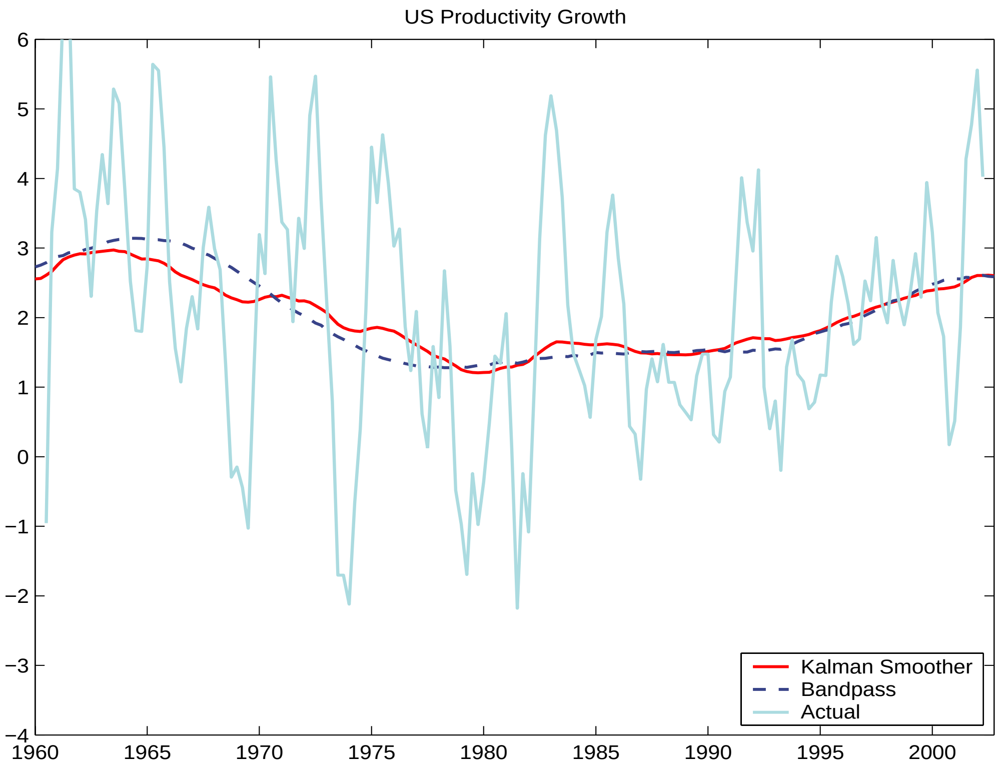
h	**USA**	**Japan**	**Germany**	**France**	**Britain**	**Canada**	**Italy**	**Neth**	**Sweden**
1	0.04	0.52	0.70	0.38	0.13	0.35	0.06	-0.29	0.72
2	0.12	0.45	0.64	0.33	0.17	0.45	-0.01	-0.15	0.76
3	0.06	0.39	0.70	0.38	0.19	0.49	0.05	-0.70	0.79
4	0.07	0.28	0.59	0.34	0.21	0.64	-0.20	-0.66	0.82
5	-0.06	0.38	0.65	0.39	0.32	0.54	-0.30	-0.79	0.84
6	0.01	0.46	0.60	0.43	0.34	0.64	0.02	-0.88	0.87
7	-0.12	0.31	0.52	0.51	0.39	0.64	0.12	-0.96	0.87
8	-0.23	0.36	0.52	0.52	0.39	0.44	-0.03	-0.89	0.89
9	-0.32	0.37	0.59	0.64	0.52	0.39	-0.03	-0.87	0.90
10	-0.49	0.45	0.64	0.70	0.54	0.50	-0.42	-0.86	0.90
Mean Diff	-0.09	0.40	0.61	0.46	0.32	0.51	-0.07	-0.70	0.84
Level	-0.81	-0.70	-0.79	-0.88	0.09	-0.89	-0.94	-0.40	0.42

FIG. 1. Productivity and the Natural Rate of Unemployment (NAIRU) Bandpass



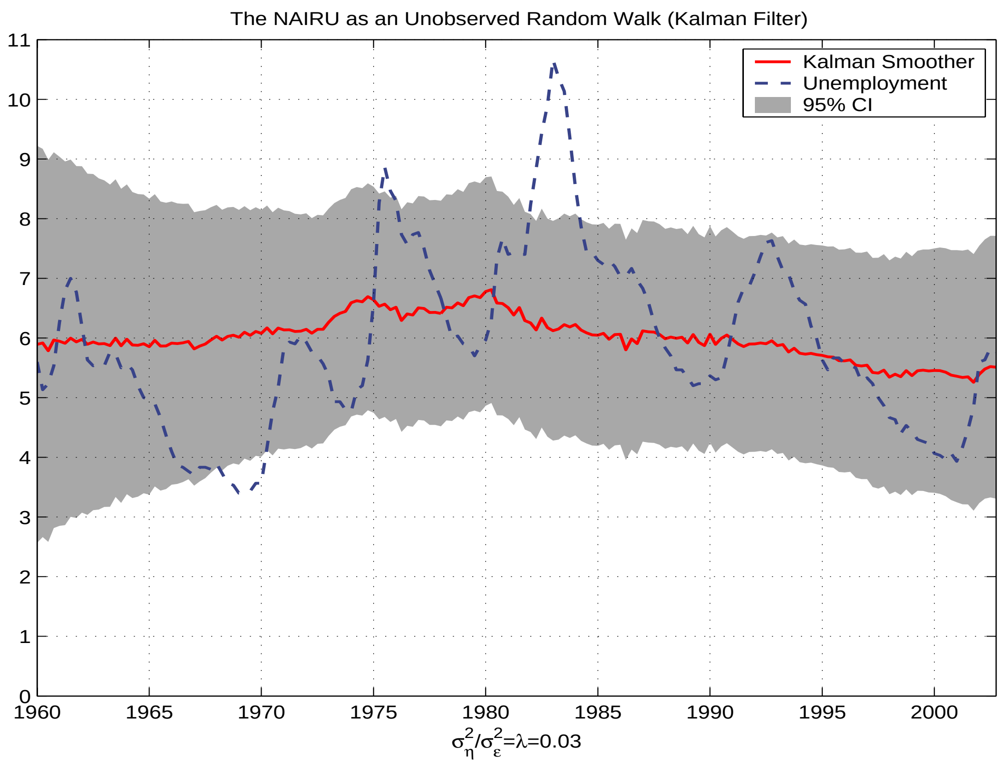
Notes: The trends are estimated using the Baxter and King (1999) bandpass filter with upper cutoff frequency of 60 quarters.

FIG. 2. Productivity Growth and Trend



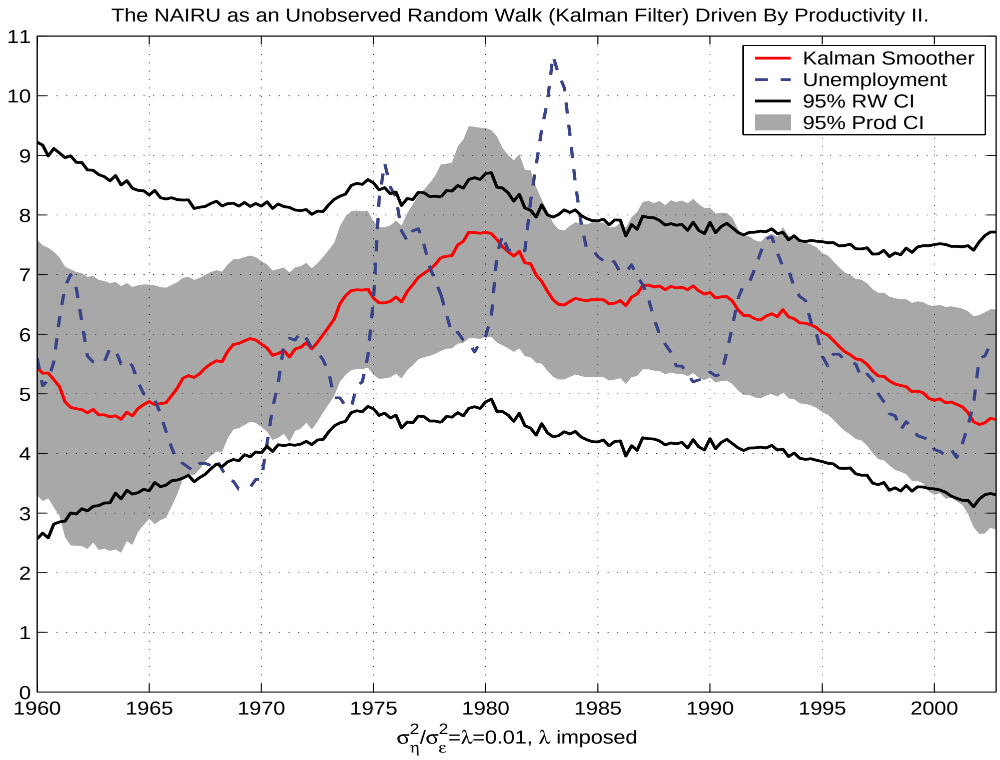
Notes: The trends are estimated using the Baxter and King (1999) bandpass filter with upper cutoff frequency of 60 quarters and Kalman smoother with the signal-to-noise ratio $\lambda_\theta = 0.005$. The actual productivity growth is year-on-year quarterly growth.

FIG. 3. Random Walk Natural Rate of Unemployment



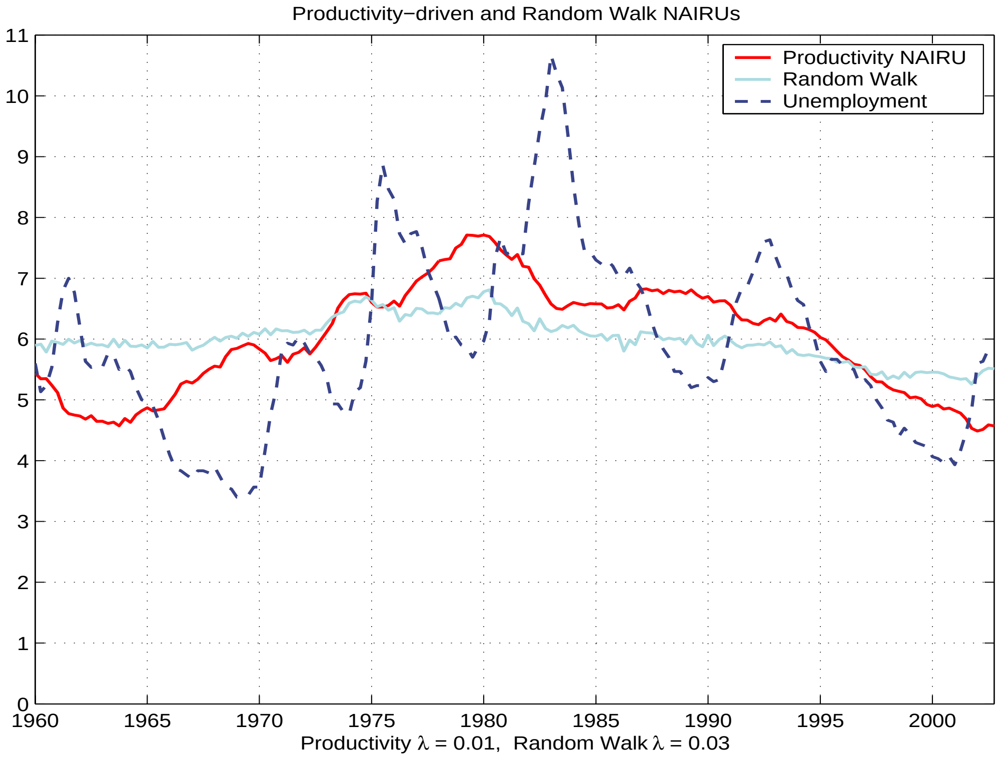
Notes: The natural rate of unemployment is estimated by the Kalman filter model (2) and assumed to follow unobserved random walk model with the signal-to-noise ratio $\lambda = 0.03$. The parameter λ is chosen to mimic the estimates of Staiger *et al.* (1997). The confidence intervals have 95% size and are obtained from the estimate of the variance of the Kalman smoother and corrected for parameter uncertainty following Ansley and Kohn (1986).

FIG. 4. Comparison of Productivity-driven and Random Walk Confidence Intervals for the NAIRU



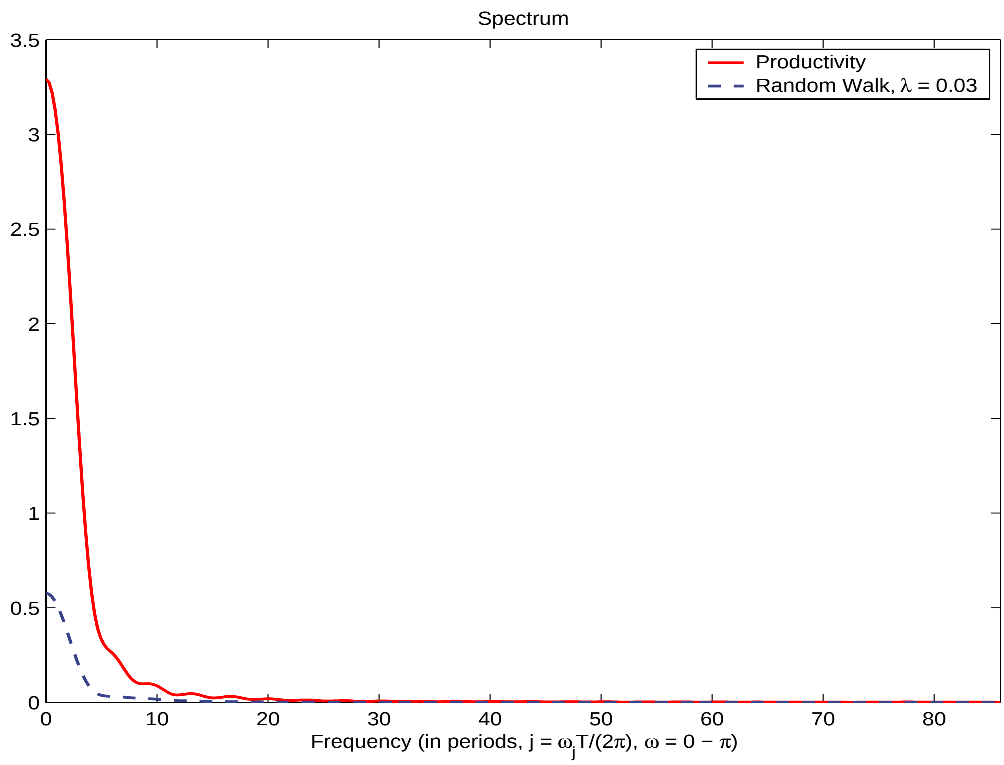
Notes: The natural rate of unemployment is estimated by the Kalman filter model (3) with the signal-to-noise ratio $\lambda = 0.01$. The confidence intervals have 95% size and are obtained from the estimate of the variance of the Kalman smoother and corrected for parameter uncertainty following Ansley and Kohn (1986).

FIG. 5. Comparison of Productivity-driven and Random Walk Natural Rates of Unemployment



Notes: The random walk natural rate of unemployment is estimated by the Kalman filter model (2) with the signal-to-noise ratio $\lambda = 0.01$. The productivity natural rate of unemployment is estimated by the Kalman filter model (3) with the signal-to-noise ratio $\lambda = 0.01$.

FIG. 6. Comparison of the Spectra of Productivity-driven and Random Walk Models



Notes: The spectra are estimated using the non-parametric estimator with the Bartlett kernel with 50 lags.

FIG. 7. Comparison of the Implied Inflation Responses to a 1% Shock to Unemployment

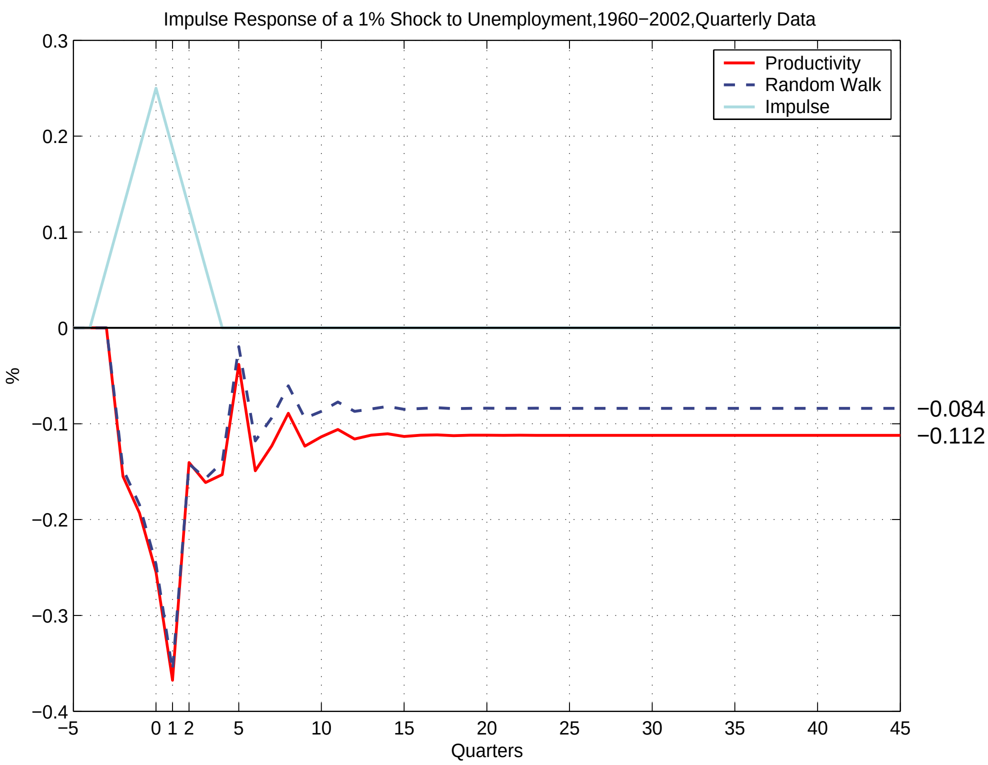


FIG. 8. Standardized Trends in Unemployment, Productivity and Change in Productivity

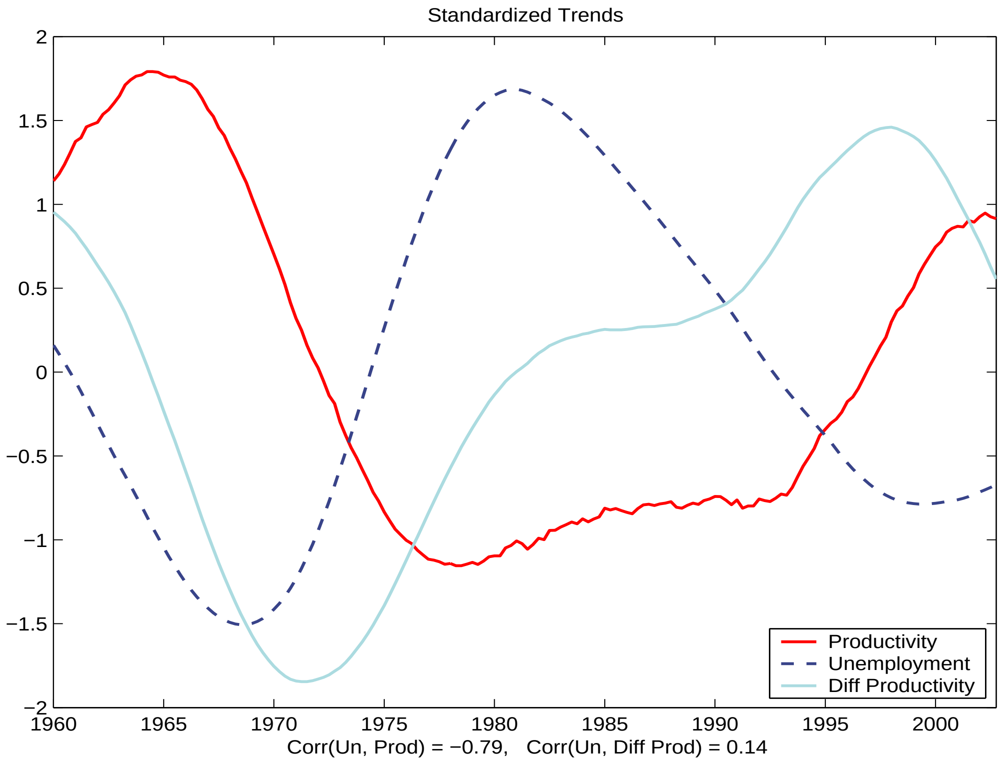


FIG. 9. Comparison of Various Signal-to-Noise Ratios, Random Walk Model

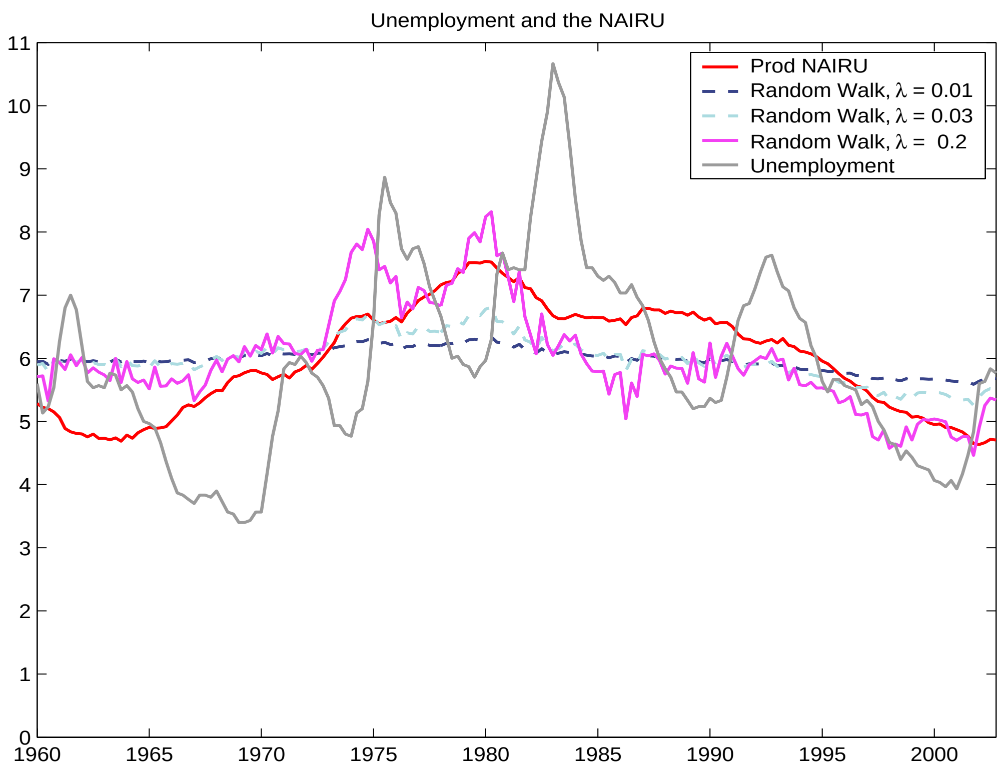


FIG. 10. Comparison of Various Signal-to-Noise Ratios, Productivity Model

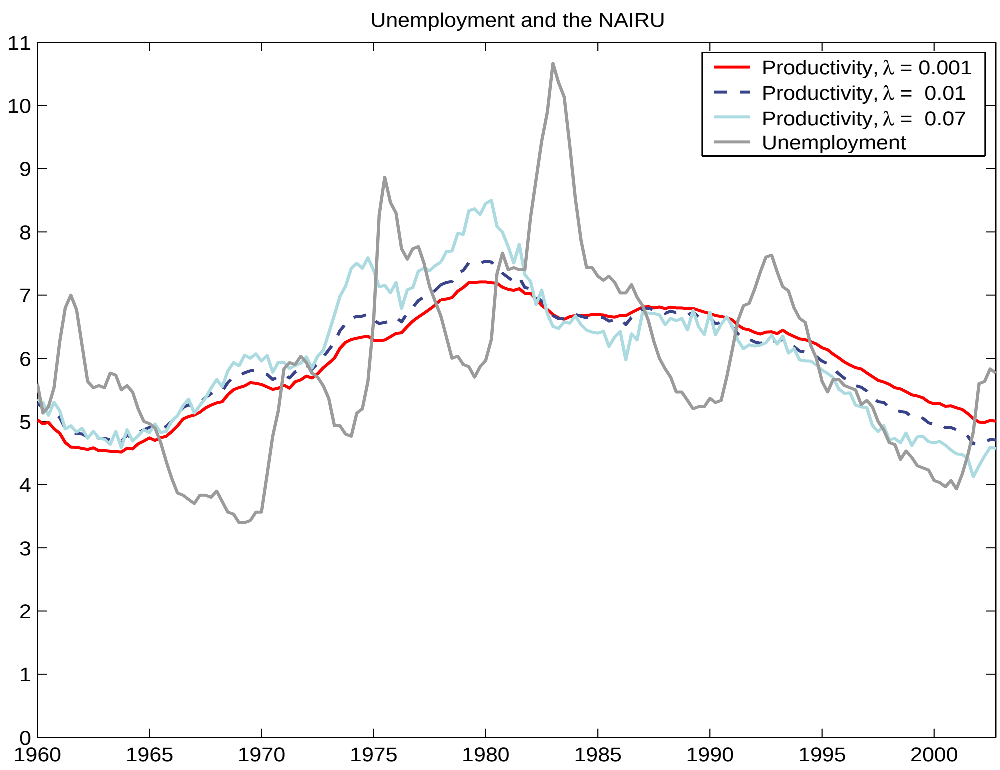


FIG. 11. Alternative Time Series

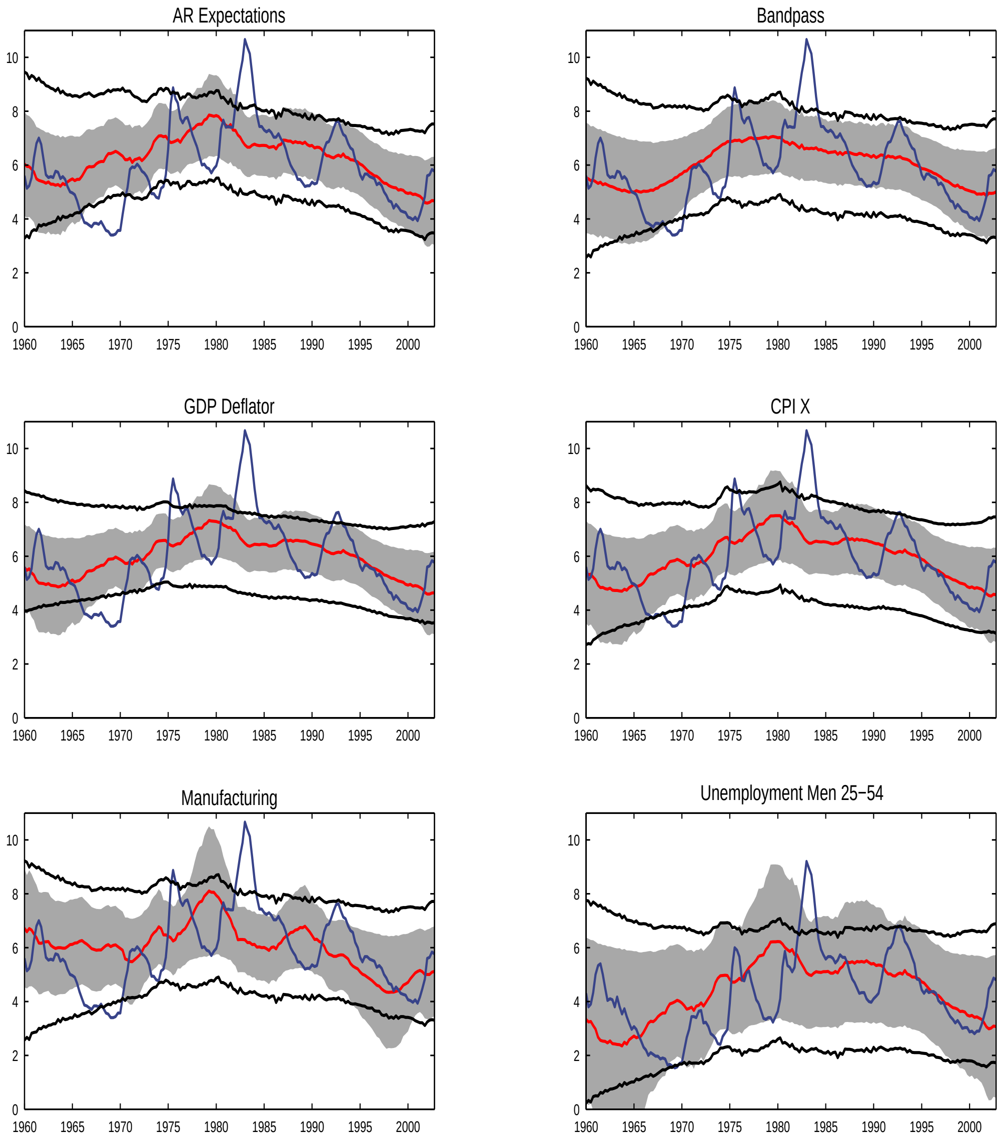
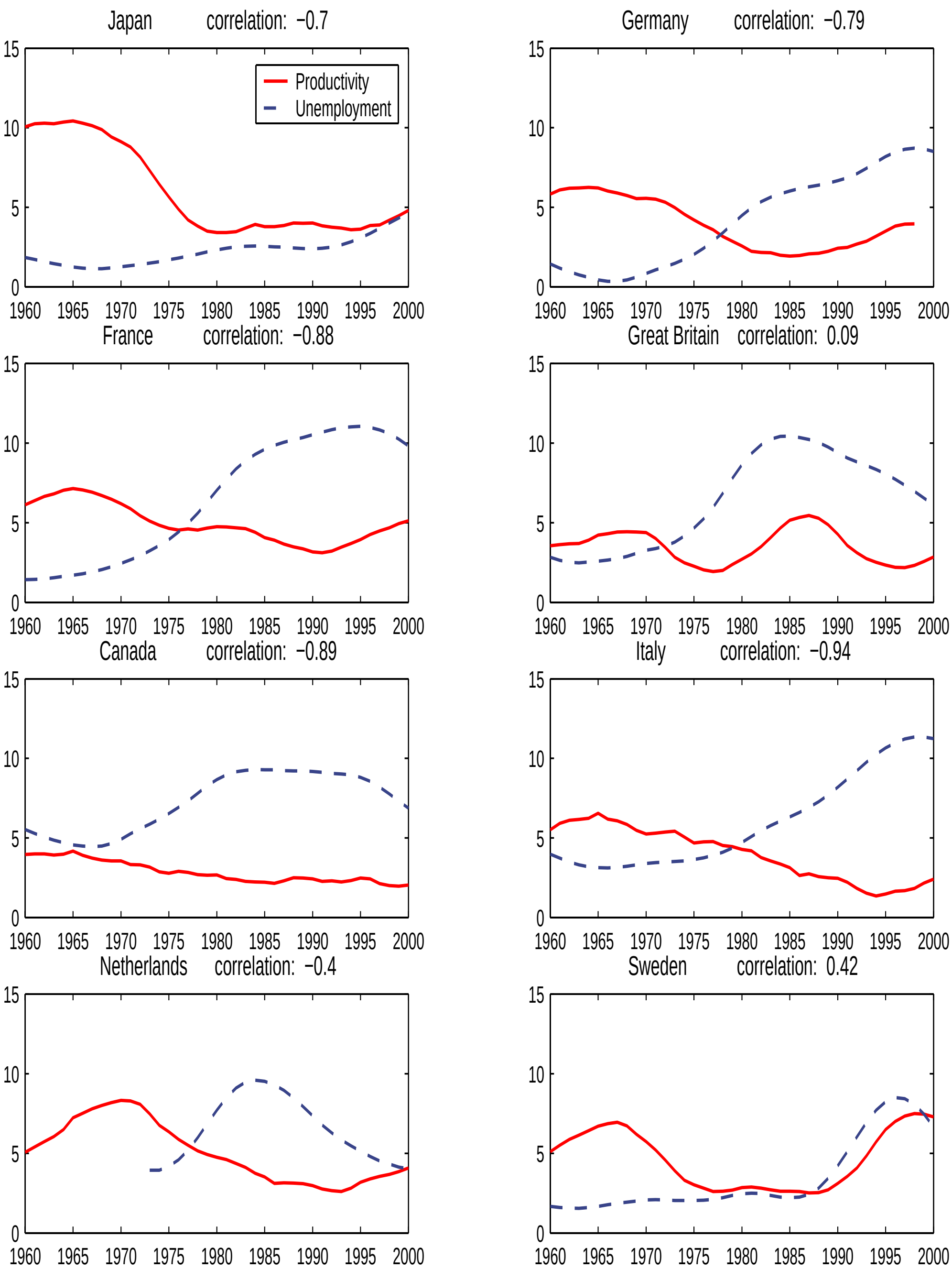


FIG. 12. International Trends in Productivity and Unemployment



Notes: The trends are estimated using the Baxter and King (1999) bandpass filter with upper cutoff frequencies of 15 years.